Writing Numbers in Columns by 25's

The activity described here should be done as soon as the child can write numbers, which was second grade for this writer and classmates.

The child takes a pad of paper or Big Chief tablet and numbers down the left side from one to twenty-five. The child then takes a ruler and draws a line down the right side of the column of numbers, and begins a new column to the right of the line, from 26-50. Another line is drawn, and a third column from 51-75 is written. This continues until 200, and stops for the day. The next day the process resumes, from 201 to 400. The third day proceeds to 600. Work should continue until the child reaches about 4000 or 5000. Much is learned and discovered by this process. Counting by 5's is reinforced, and counting by 25's and 50's is mastered.

According to graduate students in a college methods course, Problems in Teaching Remedial Mathematics, Summer 1991, this activity of numbering in columns of 25 may have these benefits to students:

1. Practice in neatness
2. Development of concentration
3. Awareness of vastness of numbers (infinite nature of numbers)
4. Practice in recitation (audible or mental: at least two students reported doing this while numbering)
5. Experience of success
6. Learning the logical progression of numbers (decimal, place-value pattern)
7. Awareness of multiples of 25, 50, 100, and 5
8. Pattern of differences-of-fives in adjacent columns

These responses were given as or after the students had completed an abbreviated version of the exercise (numbering from 1851 to 2025 per the instructions above). Below is a start for primary-grade students:

1	26	51	76	…
2	27	52	77	…
3	28	53	78	…
4	29	54	79	…
5	30	55	80	…
6	31	56	81	…
7	32	57	82	…
8	33	58	83	…
9	34	59	84	…
10	35	60	85	…
11	36	61	86	
…	…	…	…	…
…	…	…	…	…

Reading and Writing Whole Numbers and Decimal Fractions

Dr. Stan Hartzler Archer City High School

		ten-millionths
		millionths
		hundred-thousandths
		ten-thousandths
		thousandths
		hundredths
		tenths
	.	**“and”**
ones	**0**	**(or “oneths”)**
tens	**3**	
hundreds	**5**	
	,	**“thousand”**
one-	**1**	**thousand**
ten-	**4**	**thousand**
hundred-	**6**	**thousand**
	,	**“million”**
one-	**2**	**million**
ten-	**5**	**million**
hundred-	**7**	**million**
	,	**“billion”**
one-	**3**	**billion**
ten-	**6**	**billion**
hundred-	**8**	**billion**
	,	**“trillion”**
one-	**4**	**trillion**
ten-	**7**	**trillion**
hundred-	**9**	**trillion**

To read whole-number parts:

1) Read numbers between commas as groups
2) Read comma names as you find them.

Read the decimal point as “and.”

To read decimal parts:

1) Recopy numbers after decimal point without writing the point.
2) Put commas in if needed, as with a whole number, right to left.
3) Read or write the number as if it were just a whole number.
4) Finish with name of place of last digit from the decimal chart.

ROMAN NUMERALS CONNECTIONS

Dr. Stan Hartzler Archer City High School

Significance of Roman Numerals

a) There is no zero.

b) The system is not a place-value system. The Roman numeral V is always 5, no matter *where* it is; our numeral 5 could be 50 or 500 or five tenths, depending on place, hence "place value."

c) They are still in use: Superbowls, Indy Jones, old books/films.

Symbols

I = 1 V = 5 X = 10 L = 50

C = 100 D = 500 M = 1000 $\overline{\text{V}}$ = 5000

From there, any bar over a symbol means multiply by 1000. So Roman numerals are a "multiplicative" system.

Repeating Rules

I, X, C, and M may be repeated, up to three times maximum. The only exception is the clock face, which has IIII for IV, when IV may be misread. A recent newspaper account by an unreliable source claimed that a "modern" Roman Numeral system allows four repetitions to replace the subtraction rules given below. Nowhere else has such a system been mentioned. It is difficult to imagine why any modern Romans would have any interest in this.

"Subtraction-Preceding" Rules

Powers of ten (I, X, and C) may precede their multiplies-by-5-or-10 for subtraction as shown:

I may precede V (IV = 4) and I may precede X (IX = 9).
X may precede L (XL = 40) and X may precede C (XC = 90).
C may precede D (CD = 400) and C may precede M (CM = 900).

These are the ONLY times when a smaller unit may precede a larger.

Examples

IC is not 99 ; 99 = XCIX . 648 = DCXLVIII.

Numbering

I	VI	XI	XVI	XXI	XXVI	XXXI	XXXVI	XLI	XLVI
II	VII	XII	XVII	XXII	XXVII	XXXII	XXXVII	XLII	XLVII
III	VIII	XIII	XVIII	XXIII	XXVIII	XXXIII	XXXVIII	XLIII	XLVIII
IV	IX	XIV	XIX	XXIV	XXIX	XXXIV	XXXIX	XLIV	XLIX
V	X	XV	XX	XXV	XXX	XXXV	XL	XLV	L

Exercise: Roman Numeral Chart Organization

By ones, 1 to 100:

I									
II									
III									
IV									
V									
VI									
VII									
VIII									
IX									
X	**XX**	**XXX**	**XL**	**L**	**LX**	**LXX**	**LXXX**	**XC**	**C**

By tens, 1 - 1000

X									
XX									
XXX									
XL									
L									
LX									
LXX									
LXXX									
XC									
C	**CC**	**CCC**	**CD**	**D**	**DC**	**DCC**	**DCCC**	**CM**	**M**

Given the above, writing any number involves systematic combining. Every digit of the Hindu-Arabic notation is replaced by one OR MORE Roman Numeral(s).

Practice Roman Numeral versions of dates:

In what year were *Gone With the Wind* and *The Wizard of Oz* released? Write 1939 in Roman numerals. _______________

In what year was your professor born? Write 1947 in Roman Numerals. _______________

In what year was Dr. H's youngest (Molly) born? Write 1983 in Roman Numerals. _______________

Significant Digits

Dr. Stan Hartzler Archer City High School

Significant digits ***measure***, as opposed to some zeroes that are merely place-holders. Specifically:

A. One or more zeroes between non-zero digits *are* significant, because the assumption is that measurement was conducted at those places, and the measurement happened to be zero.

B. Zeroes between non-zero digits and the end of a whole number *are not* considered significant, because estimation or roundoff may have been involved.

C. Zeroes between non-zero whole-number places and a written decimal point *are* considered significant. Use of a decimal point at the end of a whole number indicates significant zeroes.

D. Zeroes between a decimal point and non-zero decimal digits are not significant, *unless* there are non-zero digits to the left of the decimal point.

E. One or more zeroes to the right of non-zero decimal digits are considered significant, which is why decimal rounding-off is so touchy.

A general principle for identifying *non*-significant zeroes:

> *non-significant zeroes would disappear if there were a more judicious choice of a measuring unit.*

For example, non-significant zeroes will probably appear if one attempts to measure the distance from Kansas City to St. Louis in millimeters.

Also, non-significant zeroes will appear if one attempts to measure the thickness of a fingernail in kilometers.

Further instructive examples are listed here:

Example	**Significant digits**
357	3
35700	3
35700.	5
3057	4
305,007,000	6
.357	3
.00357	3
.003057	4
.003570	4
.0000350700	6
20.00357	7

Significant digits are important in measurement (including accuracy and precision), and in rounding.

STAGES IN REDUCING COMMON FRACTIONS

(after students understand that there are *two ways to reduce*)

1. Manipulative activity linking unifix cubes or centicubes or paper clips, or re-assembling slices of pizza with sauce, or slices of pumpkin pie with whipped cream. See pictorial analogy below.

2. Visual analogy of the above:

$$\frac{6}{8} = \frac{0\ 0\ 0\ 0\ 0\ 0}{0\ 0\ 0\ 0\ 0\ 0\ 0\ 0} = \frac{(0\ 0)(0\ 0)(0\ 0)}{(0\ 0)(0\ 0)(0\ 0)(0\ 0)} = \frac{---}{--}$$

$$\frac{6}{9} = \frac{0\ 0\ 0\ 0\ 0\ 0}{0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0} = \frac{(0\ 0\ 0)(0\ 0\ 0)}{(0\ 0\ 0)(0\ 0\ 0)(0\ 0\ 0)} = -----$$

As the above is *separation into groups of equal size*, the process is division, a term that teacher may avoid here, or use to preview division.

3. Use of a multiplication fact table. For $\frac{6}{8}$, the student is shown a times table, and is asked, "Can you find 6 and 8 in the same row?" As both are found in the two's row, the fraction can be reduced. The reduced fraction is composed of the two numbers heading the columns above the six and eight in the two's row.

For many students, multiplication table use is easier than the division-frame and the vertical-line processes mentioned next and next.

4. Division frames: $\overline{)6}$ $\overline{)8}$

- Prerequisite: division with remainders.
- "What number can be written outside each frame and divided into both *six* and *eight* exactly?"

5. Vertical line written in front of fraction: $\left|\frac{6}{8}\right.$

Vertical line visually suggests division frames, yet is different enough to help wean students from writing division frames each time, toward simply thinking division.

6. Mental reduction of fraction by common factors.

(7) At this stage, the basic job of the elementary teacher is done. However, as a preview of algebra, prime factorization and canceling should be shown as a seventh way of reducing.

$$\frac{12}{18} = \frac{2 \bullet 2 \bullet 3}{2 \bullet 3 \bullet 3} = \frac{\cancel{2} \bullet 2 \bullet \cancel{3}}{\cancel{2} \bullet \cancel{3} \bullet 3} = \frac{?\_\_\_\_\_?}{?\_\_\_\_\_?}$$

(8) Students may make more connections via the fraction tables after some stage above -- around stages 3, 4, 5, or 6, perhaps.

Developing the Fraction Concept

Dr. Stan Hartzler Archer City High School

The notion of *fraction* has three or four aspects, depending on who is doing the writing. Four aspects will be presented here.

- ***Fractional part of a whole***. If a candy bar is cut into four equal pieces and three are eaten, then $\frac{3}{4}$ of the bar has been eaten.
- ***Fractional part of a group***. If twelve people are in equal groups at three tables, two of them square, then $\frac{2}{3}$ of the people are at square tables. No cutting is done.
- ***Ratio***, including per cent and degree measure of angles and arcs. Of every 1000 people in Sri Lanka, 512 (or 51.2%) are female. Ratios are often left unreduced. This sometimes clarifies meaning, as in 60%.
- ***Division***. The fraction $\frac{23}{4}$ equals 5.75, and division establishes the equality. Division is often expressed with a fraction bar. In this sense the ratio aspect is usually hidden ($5.25 = \frac{5.25}{1}$).

Whenever a mathematically-literate person encounters a fraction in a real-world situation (time signature in music, results of a poll, length measure), the collection of aspects of fraction meaning listed above become active in the mind as a set, as a concept node. The situation is matched with one or more of the aspects listed; understanding is obtained, and working with music, popular image, or length,is enabled.

The varied aspects of the meaning of fraction are difficult to digest. Most people who are comfortable with fractions arrived at the level of comfort with great courage and persistence, applied in many diverse situations over many years.

The typical elementary textbook exacerbates student confusion and subsequent anxiety concerning fractions. Typically, one early chapter develops the first aspect. A much later chapter develops the second aspect, without any attention to the issue of same symbolism for both a new and a vaguely-remembered, older idea. An environment has been established for student confusion and erosion of confidence.

The helpful teacher reviews the first aspect daily while the second aspect is being introduced. Confronting such probable points of confusion, together, briefly, and daily, helps develop concept nodes.

The teacher may also verbalize about the natural discomfort: “If you’re feeling confused about fractions right now, you’re all right. We’ll learn this again every day for a while, and then you’ll get used to fractions. Later, you will study two more aspects of the idea of a fraction, and you will get familiar and smart about those also.”

Addition Concept Connection:
WE ALWAYS ADD (AND SUBTRACT) LIKE KINDS

Issue	Use of Connection
Whole numbers	We don't add pigs to horses until we give them a larger name: livestock.
Common fractions	Common denominators give denominators a larger name (usually).
Decimal fractions	We line up the decimal points (NOT the right-hand edge) to add like kinds.
Algebra terms	We only add like terms. $x + x + 3x = 5x$; $2y^3 + 5y + 6y^3 = 8y^3 + 5y$, and these are unlike terms

Comparing Common Fractions and Comparing Decimal Fractions

We can use the same general strategy for comparing two or three (or more) common fractions or decimal fractions. That strategy is

1. Set the fractions up as if to add.
2. Get a common denominator.

The idea of a common denominator is not normally in our thinking when we set up decimal fractions to add. In this case we attach zeroes to the ends of decimal fractions that don't have as many digits to the right of the decimal point as the others. We haven't changed the size of the decimal fractions, and if we hurry no one will catch us violating rules about rounding and significant digits.

	.1854		.1854
SO	.2	**BECOMES**	.2000
	.186		.1860

Now there are common denominators (just read them to confirm), and comparison is easy. The *original* numbers must be written as the answers, not the forms with extra zeroes, in the required order of size.

We Always Add Like Kinds.

1. <u>Whole number addition</u>

Action: We add ones together, tens together, hundreds together, and so on. When adding ones together produces new tens, they are added with the tens.

Manifestation: We line up whole numbers right-justified to add, keeping columns neat.

2. <u>Arithmetic word problems</u>

Action: We add pigs to pigs and cows to cows.

Manifestation: We can add pigs to cows when they have a common name (animals).

3. <u>Common fraction addition</u>

Action: We add fractions that have like denominators.

Manifestation: Fractions with unlike denominators are given common denominators before addition is attempted.

4. <u>Multiplication genesis</u>

Action: We combine groups of equal size: 7 + 7 + 7 =3●7

Manifestation: 7 + 7 + 7 + 5 + 5 = 3●7 + 2●5

5. <u>Decimal fraction addition</u>

Action: We add tenths together, hundredths together, etc.

Manifestation: Lining up decimal fractions with decimal points in a row, keeping columns neat.

6. <u>Radicals</u>

Action: We can add $\sqrt{2}+\sqrt{2}+\sqrt{2}=3\sqrt{2}$

Manifestation: $2\sqrt{3}+4\sqrt{5}+3\sqrt{5}+8\sqrt{3}=10\sqrt{3}+7\sqrt{5}$

7. <u>Similar terms</u>

Action: We add $x^2+x^2+x^2=3x^2$

Manifestation: $$\left\{\begin{aligned} & x^2+x^2+x^2+x+x+x+x+x^3+x^3 \\ & \qquad = 3x^2+4x+2x^3 \end{aligned}\right.$$

Geometric Classification & Relationship

<u>Classification</u>: categorizing **one** at a time.
<u>Relationship</u>: connection between **two** things.

<table>
<tr><th>ENTITY</th><th>CLASSIFICATION</th><th>RELATIONSHIP</th><th></th></tr>
<tr><td>SETS of POINTS in one dimension</td><td>line, ray, segment</td><td>coincident, congruent</td><td rowspan="9">HOMOGENEOUS</td></tr>
<tr><td>LINES or CURVES</td><td>plane, simple, closed, simple-closed, polygon</td><td>intersecting, skew, perpendicular, parallel, coincident</td></tr>
<tr><td>ANGLES</td><td>acute, right, obtuse
straight
reflex</td><td>vertical, adjacent, corresponding, alternate interior, alternate exterior, supplementary, complementary
congruent</td></tr>
<tr><td>POLYGONS</td><td>concave, convex</td><td>congruent, similar</td></tr>
<tr><td>CONVEX Polygons</td><td>regular, irregular</td><td>congruent, similar</td></tr>
<tr><td>SOLIDS</td><td>polyhedra
prism, pyramid
other
sphere, cone, cylinder, solids of revolution</td><td>congruent
similar</td></tr>
<tr><td>TRIANGLES</td><td>sides: equilateral, isosceles, scalene
angles: acute, right, obtuse, equiangular</td><td>congruent
similar</td></tr>
<tr><td>QUADRILATERALS</td><td>square, rectangle, parallelogram, kite, rhombus, trapezoid</td><td>congruent, similar</td></tr>
<tr><td>POINTS</td><td>endpoint, midpoint, intersection, vertex</td><td></td></tr>
<tr><td>LINES/SEGMENTS in or about a circle</td><td>radius, diameter, chord, tangent, secant</td><td colspan="2" rowspan="3">HETERO–GENEOUS</td></tr>
<tr><td>ANGLES in a circle</td><td>central, inscribed</td></tr>
<tr><td>LINES, RAYS, SEGMENTS in a triangle</td><td>perpendicular bisector, altitude, median, angle bisector</td></tr>
<tr><td>ATTRIBUTES</td><td colspan="3">length, area, volume; velocity, mass, age, magnetic flux, luster, electrical charge, momentum, temperature</td></tr>
</table>

Mega-Relationships (three or more at a time):

(A) concurrent lines; (B) collinear points

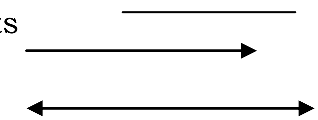
Geometric Classification/Relationship Details

"Line" <u>classification</u>

Segment -- two endpoints

Ray -- one endpoint

Line -- no endpoints



"Line" <u>relationship</u>

Intersecting -- have one or more points in common

Perpendicular -- intersect to make 90-degree angles

Parallel -- on same surface; never intersect

Coincident -- all points in common; same line; intersect everywhere

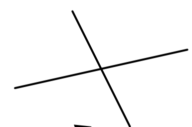
Skew -- not on same surface; never intersect; not parallel

Angle <u>classification</u>

Acute -- less than 90 degrees

Right -- equals 90 degrees

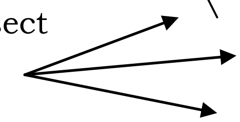
Obtuse -- bigger than 90 degrees, but less than 180 degrees



Angle <u>relationship</u>

Vertical -- across from each other when two lines intersect

Adjacent -- next to each other. Share a ray and corner (vertex) but no inside (interior) points



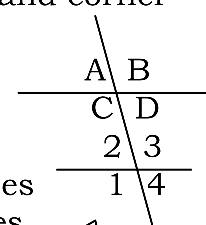
Corresponding $\angle A \& \angle 2$, etc.

Alternate interior $\angle C \& \angle 3$

Alternate exterior $\angle 1 \& \angle B$ <1 & <B

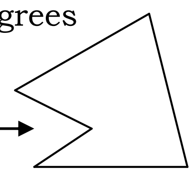
Supplementary -- measures total 180 degrees

Complementary -- measures total 90 degrees



Polygon classification

Concave -- has a "cave"



Convex -- no cave

- Regular -- all sides and angles congruent to one another
- Irregular -- not regular

<u>Polygon relationship</u>

Congruent -- same size and shape

Similar -- same shape

Solids classification

Prism -- all sides polygons; top and bottom congruent (same)

Pyramid -- all sides polygons; top is a point

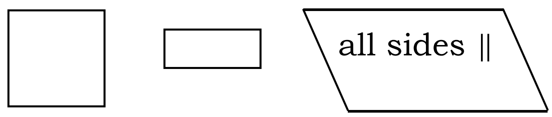
<u>Triangles classification</u>

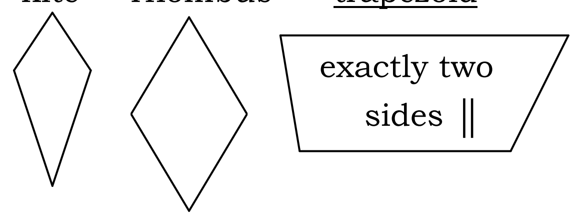
SIDES:
- Equilateral -- all *sides* the same
- Isosceles -- at least two *sides* the same
- Scalene -- no *sides* the same

ANGLES:
- Acute -- all *angles* acute
- Right -- one right *angle*
- Obtuse -- one obtuse *angle*

Quadrilateral classification

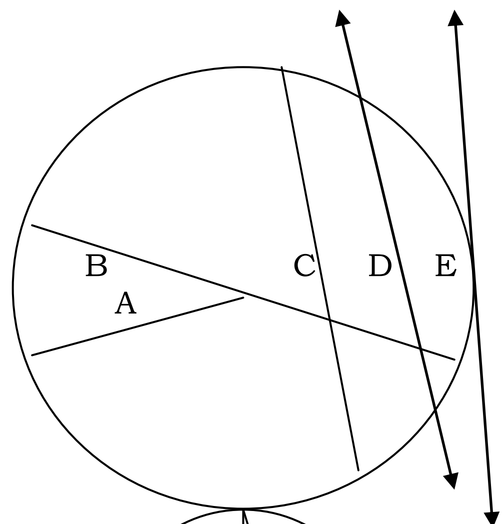
square rectangle <u>parallelogram</u> kite rhombus <u>trapezoid</u>





<u>kite</u>: adjacent sides congruent
<u>rhombus:</u> all sides congruent
<u>rectangle</u>: parallelogram with right angles <u>square</u>: rhombus with right angles

<u>Lines on or about a circle classification</u>



A radius

B diameter

C chord

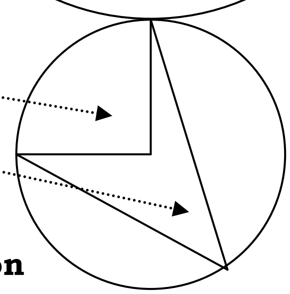
D secant

E tangent

<u>Angles in a circle classification</u>

Central: vertex is center of circle

Inscribed: vertex is a point on circle



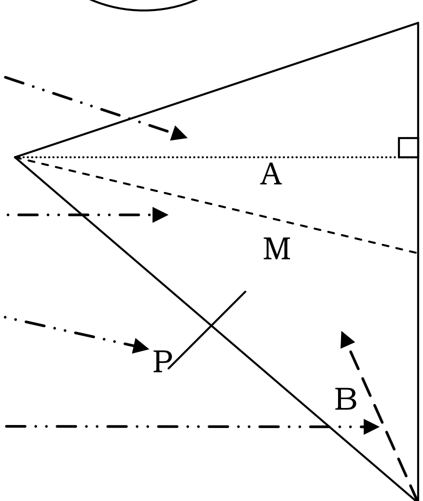
Lines/segments/rays in a triangle classification

A. Altitude: through vertex, perpendicular to opposite side.

M. Median: through vertex to midpoint of opposite side.

P. Perpendicular bisector: through midpoint of a side, perpendicular to that same side.

B. Angle bisector: through vertex, divides angle into two congruent parts.



Connection: Distance/Area/Volume

Dr. Stan Hartzler Archer City High School

Distance (length): number of segments* needed to connect two points.

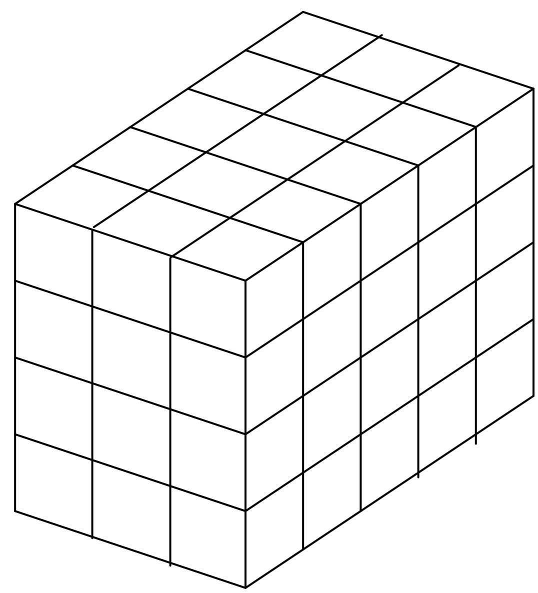
Perimeter: distance around a region

Area: number of squares* needed to cover a region or surface.

Surface Area: sum of face areas for a solid

Volume: number of cubes* needed to fill a space.

***of uniform size**



Connection activity: Show students a rectangular solid of cubes (Unifix or unit cubes, etc.), such as in the illustration:

Students are asked a set of questions that require "gearshifting" between perimeter, area, and volume, to compensate for the usual fragmented treatment of these ideas given in most textbooks.

A similar set of questions should be given daily. Such a set might be as follows:

> "What is the perimeter of the top face?"
> "What is the area of the top face?"
> "What is the volume of the entire solid?"

The same solid and questions may be used the next day, but pertaining to a different face.

Students may ask, “Why squares, not triangles? Why cubes and not pyramids?” The answer appears to be a matter of historical and multicultural practice.

CIRCLE ATTRIBUTE SCHEMA:	A	R	D	$P = C$
	_____	_____	_____	_____

Connection: Distance/Area/Volume

Distance (length): number of segments* needed to connect one point to another.

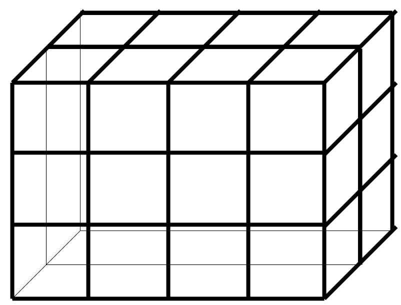
Perimeter: distance around a region

Area: number of squares* needed to cover a region or surface.

Surface Area: sum of face areas for a solid

Volume: number of cubes* needed to fill a space.

***of uniform size**



Arc Length and Sector Area Connected

For a circle with radius 12, do the circle attribute schema.

For a sector of that circle made by a central angle of 200°,

Arc Length (length of crust of apple pie slice)	**Sector Area** (Number of covering Hershey squares)
$\frac{200°}{360°} \times$ full perimeter	$\frac{200°}{360°} \times$ full area
equals what?	equals what?

Metric Units

Distance	**Area**		**Volume**	
km	km^2		km^3	
hm	hm^2	= ha	hm^3	
dkm	dkm^2	= a	dkm^3	
m	m^2		m^3	
dm	dm^2		dm^3	= L
cm	cm^2		cm^3	= mL
mm	mm^2		mm^3	
10 of each unit in one of the unit immediately above	**100** of each unit in one of the unit immediately above		**1000** of each unit in one of the unit immediately above	

Distance units in *italics* are seldom used.

Clarifying Area and Volume Units and Concepts

Once the notion is dispelled that **area** is length times width, the idea that area is found by length times width should be clarified. Finding area is often thought of as

$$3 \text{ cm} \times 4 \text{ cm} = 12 \text{ cm}^2 .$$

This is *erroneous*. Same-kind quantities are never multiplied. In the horizontal format above, the first number (*multiplier*), which is always an abstract number that tells the number of groups. The second number (*multiplicand*), carries all of the units. For the example above. four tells how many squares of area are in each row, and three tells the number of rows. In dimensional analysis format,

$$3 \text{ rows} \times \frac{4 \text{ cm}^2}{\text{row}} = 12 \text{ cm}^2$$

This thought pattern reinforces the concept of area as number of squares. This may be further reinforced with prose such as this:

We only think of multiplying length times width because

- ***number of length segments matches number of squares in each row***
- ***number of width segments matches number of rows.***

Once the notion is dispelled that **volume** is length × width × height, the idea that volume is found by length × width × height should be clarified. Finding volume is often thought of as

$$3 \text{ cm} \times (4 \text{ cm} \times 5 \text{ cm}) = 60 \text{ cm}^3 .$$

This thinking is again erroneous. Same-kind quantities are never multiplied. In the horizontal format above, the first number (*multiplier*) is always an abstract number that tells the number of groups. The second number (*multiplicand*) carries all of the units. For the example above. five tells how many cubes of volume are in each row (or "stick"), four tells how many rows or sticks per layer, and three tells how many layers. In dimensional analysis format,

$$3 \text{ layers} \times \left(\frac{4 \text{ rows}}{\text{layer}} \times \frac{5 \text{ cm}^3}{\textit{row}} \right) = 60 \text{ cm}^3$$

This thought pattern reinforces the concept of volume as number of cubes. This may be further reinforced with prose such as this:

We only think of multiplying length × width × height because

- ***number of length segments matches number of cubes in each row***
- ***number of width segments matches number of rows in each layer***
- ***number of height segments matches number of layers.***

Talton's
Thinking Approach to Problem-Solving (TAPS)

<u>An Approach to Helping Students with</u>

Arithmetic Word Problems

The following outline should be on a classroom poster, with "combine" etc. explained initially with manipulatives. This introduction must be repeated at least briefly for several consecutive days.

Thereafter, students read (or have read to them) two or three varied word problems each day. "Varied" means that not all of the problems are solved using the same operation or combination thereof. (When a textbook page is headed by "Using Addition in Problem-Solving", no problem-solving will transpire -- decisions will not be made by students, many of whom will not even read the problems, but will add whatever numbers appear therein.) A Saxon/Hake/Larson book is the ideal here.

When students can't decide, the teacher refers them to this chart, and reminds them with manipulatives if the chart alone doesn't help.

Does the problem suggest that we…

I. <u>Combine</u> groups?
   A. If groups are of equal size, multiply.*
   B. Otherwise, add.

II. <u>Separate</u> a group?
   A. If parts of the group are equal, divide.*
   B. Otherwise, subtract.

III. <u>Compare</u> two groups?
   A. If to find a difference, subtract.*
   B. If to find a ratio, divide.

*Eliminate these for primary-grade students as needed.

<u>Advanced Version Showing Properties</u>

	Combining ⇩	Separating (comparing) ⇩	
(Inverses) →	Addition	Subtraction ⇦ (comparing to find differences)	GROUPS OF <u>ANY</u> SIZE
(Inverses) →	Multiplication	Division ⇦ (comparing to find ratios)	GROUPS OF <u>EQUAL</u> SIZE
	Properties: ⇧ associative commutative identity	No standard properties ⇧ one-sided identity statemental commutativity* * $8 \div 4 = 2 \Leftrightarrow 8 \div 2 = 4$	
	Distributive property of × or ÷ over + or –		

Algebraic Thinking From Arithmetic

$4 + 6 = 6 + 4$

$11 + 8 = 8 + 11$

$x + y =$ _________

$7 + (2 + 13) = (7 + 2) + 13$

$(3 + 8) + 5 = 3 + (8 + 5)$

$a + (b + c) =$ _____________

$5 \times (6 + 2) = 5\times6 + 5\times2$

$5 \times (11 + 10) = 5\times11 + 5\times10$

$5 \times (5 + 9) = 5\times5 + 5\times9$

$5 \times (a + b) =$ ____________

$\frac{4+6}{6+4} =$ ____ $\quad \frac{30+20}{20+30} =$ ____ $\quad \frac{9+14}{14+9} =$ _____ $\quad \frac{d+f}{?} =$ _____

$\frac{9}{12} = \frac{15}{20} \Rightarrow \frac{12}{20} = \frac{9}{15}$

$\frac{5}{6} = \frac{10}{12} \Rightarrow \frac{6}{12} = \frac{5}{10}$

$\frac{3}{5} = \frac{9}{15} \Rightarrow \frac{5}{15} = \frac{3}{9}$

$\frac{x}{r} = \frac{z}{b} \Rightarrow ?$

If $7 > 4$ and $4 > x$, then

$7 > x$

If $10 > b$ and $b > 1$, then

$10 > 1$

If $x > a$ and $a > d$, then

? __________ ?

$4 > 1$, so $1 < 4$

$8 > -2$, so $-2 < 8$

If $c > a$, then _______

$\frac{2}{5} = \frac{2+2}{5+5} = \frac{2+2+2}{5+5+5} = \ldots$

$\frac{6}{7} = \frac{6+6}{7+7} = \frac{6+6+6}{7+7+7} = \ldots$

$\frac{n}{p} = ?$

$$\begin{array}{r} 12\frac{3}{7} \\ -4\frac{5}{7} \\ \hline \end{array} = \begin{array}{r} 11 + 1\frac{3}{7} = 11\frac{10}{7} \\ -4\frac{5}{7} \\ \hline \end{array}$$

$$\begin{array}{r} 20\frac{5}{9} \\ -10\frac{8}{9} \\ \hline \end{array} = \begin{array}{r} 19 + 1\frac{5}{9} = 19\frac{14}{9} \\ -10\frac{8}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 7\frac{7}{x} \\ -2\frac{9}{x} \\ \hline \end{array} = ?$$

$(x^3 \bullet y^2)^4 = x^{12} \bullet y^8$

$(a^4 \bullet z^5)^2 = a^8 \bullet z^{10}$

$(d^2 e^4 m)^3 = d^6 e^{12} m^3$

$(c^x f^b)^5 = ?$

$4 + 4 + 4 + 4 + 4 = \_\_\_\_ \times 4$ $3 + 3 + 3 + 3 + 3 = \_\_\_\_ \times 3$ $x + x + x + x + x = \_\_\_\_ \times x$	$\frac{7 \bullet 11}{3 \bullet 11} =$ $\frac{6 \bullet 5}{7 \bullet 5} =$ $\frac{x \bullet y}{x \bullet z} =$	$\frac{3}{7} + \frac{2}{7}$	$\frac{4}{11} + \frac{6}{11}$	$\frac{3}{z} + \frac{5}{z}$
$\frac{4-6}{6-4} = \_\_\_\_$	$\frac{30-20}{20-30} = \_\_\_\_$	$\frac{9-14}{14-9} = \_\_\_\_$	$\frac{x-y}{?} = \_\_\_\_$	

Algebra word problems are rarely approached from the standpoint of arithmetic, and such an approach should be considered. Students are thrown into several types of word problems with almost no preview of the background arithmetic situations. These word problem types include

Consecutive integers	Angles	Coin
Distance-rate-time	Rate-of-work	Digit
Per cent mixture	Chemical mixture	

An arithmetic background might begin with problems such as these in the assignments several days prior to the standard problems:

For *consecutive integers*:

A. Integers are elements of the set {..., –3, –2, –1, 0, 1, 2, 3, ...} Three consecutive integers are 19,20,21. Four consecutive odd integers are 33,35,37,39. Write the five consecutive even integers that precede 43.

Now: Represent three consecutive integers if the first is y.
Represent three consecutive odd integers if the second is y.
Represent three consecutive even integers if the last is z.

For *angle problems*:

B. Two complementary angles have sum of 90°; two supplementary angles have sum 180°. Find the measure of an angle that is complementary to a 34° angle; then find the supplement of the same angle.

Now: What is the complement of an $x°$ angle?
What is the supplement of a $y°$ angle?

For *distance-rate-time*:

C. How far does a plane travel in three and one-half hours at a rate of 460 miles per hour? How long does it take a cyclist to go 91 miles at 13 miles per hour?

If Bugs heads off at 14 mph and Daffy travels 12 mph in the opposite direction, how far apart are they in 3 hours? In x hours?

Now: If Shirk heads off at a mph and Shrieka heads away at b mph in the opposite direction, how far apart are they in d hours?

Sample Spaces

Dr. Stan Hartzler Archer City High School

A sample space for a situation involving different possible outcomes is the set of all possible outcomes for that situation. Verifying correctness of a sample space is a matter of experimenting, as with rolling two dice.

Following is a sample of sample spaces. Examination and then use of these sample spaces should help develop the sample space concept.

One Coin
H
T

Two Coins	
H	**H**
H	**T**
T	**H**
T	**T**

Three Coins		
H	**H**	**H**
H	**H**	**T**
H	**T**	**H**
H	**T**	**T**
T	**H**	**H**
T	**H**	**T**
T	**T**	**H**
T	**T**	**T**

Four Coins			
H	**H**	**H**	**H**
H	**H**	**H**	**T**
H	**H**	**T**	**H**
H	**H**	**T**	**T**
H	**T**	**H**	**H**
H	**T**	**H**	**T**
H	**T**	**T**	**H**
H	**T**	**T**	**T**
T	**H**	**H**	**H**
T	**H**	**H**	**T**
T	**H**	**T**	**H**
T	**H**	**T**	**T**
T	**T**	**H**	**H**
T	**T**	**H**	**T**
T	**T**	**T**	**H**
T	**T**	**T**	**T**

Five Coins				
H	**H**	**H**	**H**	**H**
H	**H**	**H**	**H**	**T**
H	**H**	**H**	**T**	**H**
H	**H**	**H**	**T**	**T**
H	**H**	**T**	**H**	**H**
H	**H**	**T**	**H**	**T**
H	**H**	**T**	**T**	**H**
H	**H**	**T**	**T**	**T**
H	**T**	**H**	**H**	**H**
H	**T**	**H**	**H**	**T**
H	**T**	**H**	**T**	**H**
H	**T**	**H**	**T**	**T**
H	**T**	**T**	**H**	**H**
H	**T**	**T**	**H**	**T**
H	**T**	**T**	**T**	**H**
H	**T**	**T**	**T**	**T**
T	**H**	**H**	**H**	**H**
T	**H**	**H**	**H**	**T**
T	**H**	**H**	**T**	**H**
T	**H**	**H**	**T**	**T**
T	**H**	**T**	**H**	**H**
T	**H**	**T**	**H**	**T**
T	**H**	**T**	**T**	**H**
T	**H**	**T**	**T**	**T**
T	**T**	**H**	**H**	**H**
T	**T**	**H**	**H**	**T**
T	**T**	**H**	**T**	**H**
T	**T**	**H**	**T**	**T**
T	**T**	**T**	**H**	**H**
T	**T**	**T**	**H**	**T**
T	**T**	**T**	**T**	**H**
T	**T**	**T**	**T**	**T**

Two Dice

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

POKER DECK

	A	2	3	4	5	6	7	8	9	10	J	Q	K
H													
S													
D													
C													

Tree Diagrams and Probability

- When are probability values multiplied? ("and" ; $\times$; set *intersection*)
- When are probability values added? ("or" ; $+$; set *union*)

Example: A jar contains three gold marbles, two black marbles, and two white ones. A marble is drawn out and not replaced. A second marble is then drawn out. What is the probability that both are gold?

I. Draw the tree diagram.

A. What experiment does the first stage represent?

B. What experiment does the second stage represent? Be sure to specify replacement or no replacement.

C. How is the probability for each final outcome calculated? Why?

D. What is the probability that at least one white marble will be drawn?

E. How was this probability calculated? Why?

F. What is the sample space for this experiment?

G. What is the probability that no white marble will be drawn?

H. Are there complementary events exemplified above? If *yes*, which questions are involved?