Clarifying Area and Volume Units and Concepts

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Once the notion is dispelled that **area** is length times width, the idea that area is found by length times width should be clarified. Finding area is often thought of as

$$3 \text{ cm} \times 4 \text{ cm} = 12 \text{ cm}^2 .$$

This thinking is *erroneous*, however. Same-kind quantities are never multiplied. In the horizontal format above, the first number is the multiplier, which is always an abstract number that tells the number of groups. The second number, the multiplicand, carries all of the units. For the example above. four tells how many squares of area are in each row, and three tells the number of rows. In dimensional analysis format,

$$3 \text{ rows} \times \frac{4 \text{ cm}^2}{\text{row}} = 12 \text{ cm}^2$$

This thought pattern reinforces the concept of area as number of squares. This may be further reinforced with prose such as this:

We only think that we multiply length times width because

- *number of length segments matches number of squares in each row*
- *number of width segments matches number of rows.*

Once the notion is dispelled that **volume** is length × width × height, the idea that volume is found by length × width × height should be clarified. Finding volume is often thought of as

$$3 \text{ cm} \times (4 \text{ cm} \times 5 \text{ cm}) = 60 \text{ cm}^3 .$$

This thinking is again erroneous. Same-kind quantities are never multiplied. In the horizontal format above, the first number is the multiplier, which is always an abstract number that tells the number of groups. The second number, the multiplicand, carries all of the units. For the example above. five tells how many cubes of volume are in each row (or "stick"), four tells how many rows or sticks per layer, and three tells how many layers. In dimensional analysis format,

$$3 \text{ layers} \times \left(\frac{4 \text{ rows}}{\text{layer}} \times \frac{5 \text{ cm}^3}{row} \right) = 60 \text{ cm}^3$$

This thought pattern reinforces the concept of volume as number of cubes. This may be further reinforced with prose such as this:

We only think that we multiply length × width × height because

- *number of length segments matches number of cubes in each row*
- *number of width segments matches number of rows in each layer*
- *number of height segments matches number of layers.*