

Algebraic Thinking From Arithmetic

Dr. Stan Hartzler Archer City High School

$4 + 6 = 6 + 4$	$7 + (2 + 13) = (7 + 2) + 13$	$5 \times (6 + 2) = 5 \times 6 + 5 \times 2$
$11 + 8 = 8 + 11$	$(3 + 8) + 5 = 3 + (8 + 5)$	$5 \times (11 + 10) = 5 \times 11 + 5 \times 10$
$x + y = \underline{\hspace{2cm}}$	$a + (b + c) = \underline{\hspace{2cm}}$	$5 \times (5 + 9) = 5 \times 5 + 5 \times 9$
$5 \times (a + b) = \underline{\hspace{2cm}}$		
$\frac{4+6}{6+4} = \underline{\hspace{1cm}}$	$\frac{30+20}{20+30} = \underline{\hspace{1cm}}$	$\frac{9+14}{14+9} = \underline{\hspace{1cm}}$
		$\frac{d+f}{?} = \underline{\hspace{1cm}}$
$\frac{9}{12} = \frac{15}{20} \Rightarrow \frac{12}{20} = \frac{9}{15}$	If $7 > 4$ and $4 > x$, then $7 > x$	$4 > 1$, so $1 < 4$
$\frac{5}{6} = \frac{10}{12} \Rightarrow \frac{6}{12} = \frac{5}{10}$	If $10 > b$ and $b > 1$, then $10 > 1$	$8 > -2$, so $-2 < 8$
$\frac{3}{5} = \frac{9}{15} \Rightarrow \frac{5}{15} = \frac{3}{9}$	If $x > a$ and $a > d$, then $? \underline{\hspace{1cm}} ?$	If $c > a$, then $\underline{\hspace{2cm}}$
$\frac{x}{r} = \frac{z}{b} \Rightarrow ?$		$\frac{2}{5} = \frac{2+2}{5+5} = \frac{2+2+2}{5+5+5} = \dots$
		$\frac{6}{7} = \frac{6+6}{7+7} = \frac{6+6+6}{7+7+7} = \dots$
		$\frac{n}{p} = ?$
$\begin{array}{r} 12\frac{3}{7} \\ -4\frac{5}{7} \\ \hline \end{array} = \begin{array}{r} 11+1\frac{3}{7} = 11\frac{10}{7} \\ -4\frac{5}{7} \\ \hline \end{array}$	$(x^3 \bullet y^2)^4 = x^{12} \bullet y^8$	
$\begin{array}{r} 20\frac{5}{9} \\ -10\frac{8}{9} \\ \hline \end{array} = \begin{array}{r} 19+1\frac{5}{9} = 19\frac{14}{9} \\ -10\frac{8}{9} \\ \hline \end{array}$	$(a^4 \bullet z^5)^2 = a^8 \bullet z^{10}$	
$\begin{array}{r} 7\frac{7}{x} \\ -2\frac{9}{x} \\ \hline \end{array} = ?$	$(d^2 e^4 m)^3 = d^6 e^{12} m^3$	
	$(c^x f^b)^5 = ?$	

$4 + 4 + 4 + 4 + 4 = \underline{\hspace{1cm}} \times 4$	$\frac{7 \bullet 11}{3 \bullet 11} =$	$\frac{3}{7}$	$\frac{4}{11}$	$\frac{3}{z}$
$3 + 3 + 3 + 3 + 3 = \underline{\hspace{1cm}} \times 3$	$\frac{6 \bullet 5}{7 \bullet 5} =$	$+\frac{2}{7}$	$+\frac{6}{11}$	$+\frac{5}{z}$
$x + x + x + x + x = \underline{\hspace{1cm}} \times x$	$\frac{x \bullet y}{x \bullet z} =$	$\underline{\hspace{1cm}}$	$\underline{\hspace{1cm}}$	$\underline{\hspace{1cm}}$
$\frac{4-6}{6-4} = \underline{\hspace{1cm}}$	$\frac{30-20}{20-30} = \underline{\hspace{1cm}}$	$\frac{9-14}{14-9} = \underline{\hspace{1cm}}$	$\frac{x-y}{?} = \underline{\hspace{1cm}}$	

Algebra word problems are rarely approached from the standpoint of arithmetic, and such an approach should be considered. Students are thrown into several types of word problems with almost no preview of the background arithmetic situations. These word problem types include

Consecutive integers

Angles

Coin

Distance-rate-time

Rate-of-work

Digit

Per cent mixture

Chemical mixture

An arithmetic background might begin with problems such as these in the assignments several days prior to the standard problems:

For consecutive integers:

A. Integers are elements of the set $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

Three consecutive integers are 19,20,21. Four consecutive odd integers are 33,35,37,39. Write the five consecutive even integers that precede 43.

Now: Represent three consecutive integers if the first is y .

Represent three consecutive odd integers if the second is y .

Represent three consecutive even integers if the last is z .

For angle problems:

B. Two complementary angles have sum of 90° ; two supplementary angles have sum 180° . Find the measure of an angle that is complementary to a 34° angle; then find the supplement of the same angle.

Now: What is the complement of an x° angle?

What is the supplement of a y° angle?

For distance-rate-time:

C. How far does a plane travel in three and one-half hours at a rate of 460 miles per hour? How long does it take a cyclist to go 91 miles at 13 miles per hour?

If Bugs heads off at 14 mph and Daffy travels 12 mph in the opposite direction, how far apart are they in 3 hours? In x hours?

Now: If Shirk heads off at a mph and Shrieka heads away at b mph in the opposite direction, how far apart are they in d hours?