WHERE DID THAT ***ONE*** COME FROM?

Dr. Stan Hartzler Archer City High School

The number one appears from out of nowhere, or so it seems, in a variety of places in algebra. The concept of "default value" certainly applies. Providing a list such as the following may be helpful to students:

$$a + 4a = 1a + 4a = 5a$$

$$a \bullet a^3 = a^1 a^3 = a^4 \quad \text{and} \quad \frac{a^{10}}{a} = \frac{a^{10}}{a^1} = a^9$$

$$2x + 2 = 2x + 2 \bullet 1 = 2(x + 1) \quad \text{and} \quad y^2 + y = y^2 + 1 \bullet y = y(y + 1)$$

$$\frac{3}{3x+6} = \frac{1 \bullet 3}{3(x+2)} = \frac{1}{x+2} \quad \text{and} \quad \frac{a^3}{a^5} = \frac{1 \bullet a^3}{a^5} = \frac{1}{a^2}$$

$$\frac{a}{3} + b = \frac{a}{3} + \frac{b}{1} = \quad \cdots$$

Algebra: Addends & Products & Powers

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As a high school freshman in 1961, this writer had trouble with much of the basic simplification material that follows. Listing a few instructive examples inside the back cover of his Algebra One textbook was extremely helpful. The list grew as the course progressed.

The writer's students are encouraged to paste a list like this inside the back covers of their algebra textbooks, or to write a similar list, for quick reference. Several weeks of getting things right is often needed before independence is obtained.

Addends	**Products**	**Powers**
$3+3+3+3=4\bullet 3$ $\sqrt{2}+\sqrt{2}+\sqrt{2}+\sqrt{3}=3\sqrt{2}+\sqrt{3}$ $x+x+x+y=3x+y$ $y^2+y^2+y^2+y^4=3y^2+y^4$ $3z^2+2z^2=5z^2$	$3\bullet 3\bullet 3\bullet 3=3^4=81$ $\sqrt{2}\sqrt{2}=2$ $\sqrt{2}\sqrt{3}=\sqrt{6}$ $a^3\bullet a^2=a^5$ $3w^3\bullet 2w^2=6w^5$	$a^2\ ^3=a^6$ $\sqrt{2}\ ^2=2$ $x^{\frac{3}{4}}=\sqrt[4]{x}\ ^3$ $x^{-1}=\frac{1}{x}$ $y^{-2}=\frac{1}{y^2}$ $3p^2\ ^4=81p^8$

Quadratic Formula Development and Graph

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When you can't factor a ________________ ____________________ like $y = f(x) = 2x^2 + 3x + 2 = 0$,

(A) complete the square, *which no one does outside of courses.* **(B)** graph; look for __________________

(C) use the quadratic formula: Given $y = f(x) = ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

steps	$y = f(x) = 2x^2 + 3x + 2 = 0$	$y = f(x) = ax^2 + bx + c = 0$
1. Isolate the terms containing x on the left, with spaces. Non-x constant number must be alone on the right.	1	
2. If coefficient of x^2 is not one, divide each term by coefficient of x^2.	2,4,5	
3. Set up completion of square structure: parentheses and exponent, with x, + or −, and half of x coefficient		
4. Complete square above structure with (half of x coefficient)^2, added to both sides.	3,5	
5. Add the terms on the right side.	6	
6. Write the square root of both sides, using a ± symbol in front of the radical sign on the right side.	7	
7. Solve for x.		

1. Solutions/roots/zeroes/answers:

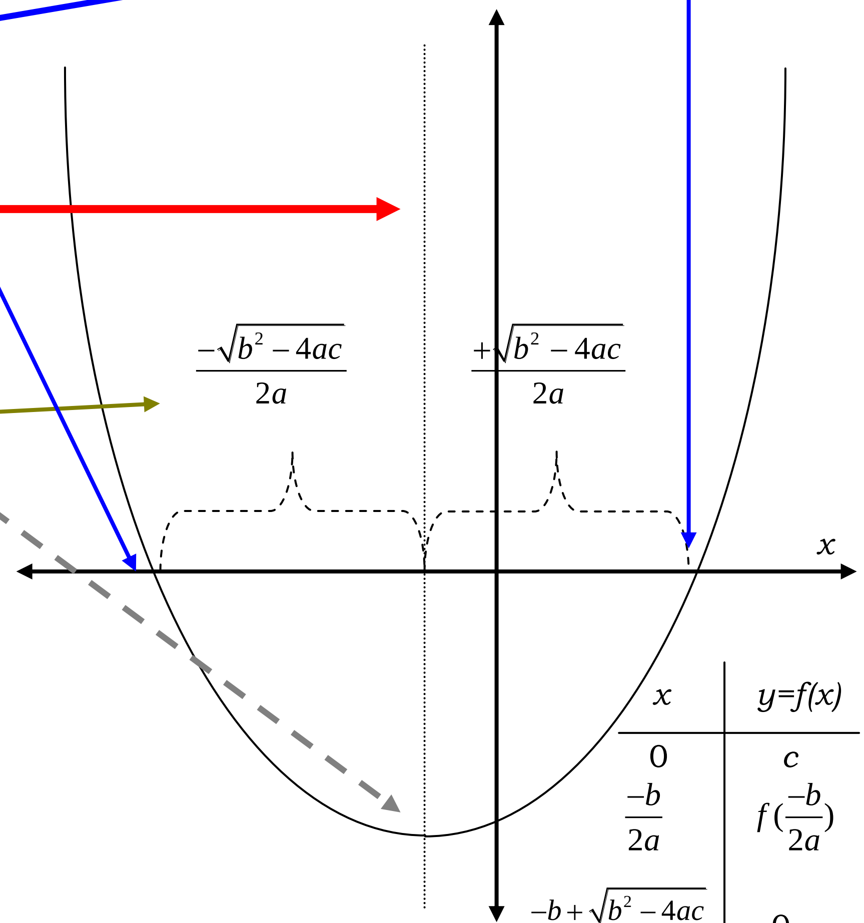
If $a > 0$, $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$

2. Axis of symmetry: $x = \dfrac{-b}{2a}$

3. Vertex: $\left(\dfrac{-b}{2a}, f\left(\dfrac{-b}{2a}\right)\right)$

4. Discriminant: $\sqrt{b^2 - 4ac}$

- If **Discriminant = 0**, then the "two" solutions are equal to each other – **ONE solution** in reality. The "two" intersections with the x axis are ONE in reality and are the vertex.
- If **Discriminant < 0**, then the "two" solutions are imaginary – **NO real solutions**. There are no intersections with the x axis.
- If **Discriminant > 0**, then there really are **TWO unequal real** solutions, and they differ because of the $\pm$ choice. The $x = \dfrac{-b}{2a}$ symmetry value is an average of the two roots therefore.



x	$y=f(x)$
0	c
$\dfrac{-b}{2a}$	$f(\dfrac{-b}{2a})$
$\dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$	0
$\dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$	0

Notes:

- If $a < 0$, the whole parabola gets turned upside down.

$$ax^2 + bx + c = 0$$

$$ax^2 + bx + \quad = -c$$

$$\frac{ax^2}{a} + \frac{bx}{a} + \quad = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}\frac{\bullet 4a}{\bullet 4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$-\frac{b}{2a} = -\frac{b}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For a quiz or exam, one may omit the second line and still get full credit.

Rational Functions Extravaganza

If the numerator's highest <u>exponent</u> is greater than that of the denominator, the graph line **"ends"** go to $y = \pm\infty$ and there is NO **horizontal symptote**.

If the numerator's highest exponent is ONE greater than the denominator's highest, then a slant asymptote exists and is found by polynomial division.*

If the numerator's highest <u>exponent</u> is equal to that of the denominator, the **"ends"** of the graph line(s) go to a **horizontal asymptote**, the equation of which is $y = \frac{6}{2} = 3$ in this case.

$$y = \frac{6x^4 + 2x^2 - x + 10}{3x^3 - 4x + 5}$$

$$y = \frac{6x^3 + 2x^2 - x + 12}{2x^3 - 9x - 4}$$

$$y = \frac{8x^3 + 2x^2 - x + 6}{2x^5 - 4x^3 + 2}$$

***y*-intercepts**: y value when $x = 0$
(Link to $y = mx + b$)

***x*-intercepts**: x value(s) when $y = 0$.
These are "roots" "solutions" "answers" "zeroes"
These ***x* -axis intercepts** are values of x that make the <u>numerator</u> equal zero.

Values of x that make the <u>denominator</u> equal zero (1) are forbidden, and (2) are x -values that determine **vertical asymptotes**.

If the numerator's highest <u>exponent</u> is less than that of the denominator, the **"ends"** of the graph line go to $y = 0$, the x axis, which is the **horizontal asymptote**.

*Slant asymptote here:

Equation of slant asymptote is $y = 2x + 4$y

$$2x + 4 + \frac{5x - 10}{3x^3 - 4x + 5}$$

$$3x^3 - 4x + 5 \overline{)\,6x^4 + 0x^3 + 4x^2 - x + 10}$$

$$\underline{6x^4 + 0x^3 - 8x^2 + 10x}$$

$$12x^2 - 11x + 10$$

$$\underline{12x^2 - 16x + 20}$$

$$5x - 10$$

<u>Grandson of Dilbert</u>

A Study of a Partially Disassembled Prism

Each ear is a regular hexagon.

The angle bisectors of a regular hexagon divide the regular* hexagon into six congruent equilateral triangles. <u>Draw the angle bisectors</u>.

***What <u>is</u> a regular polygon?**

Each side of the hexagon is 3 cm.

For an equilateral triangle with side s,

the area is $\frac{s^2\sqrt{3}}{4}$.

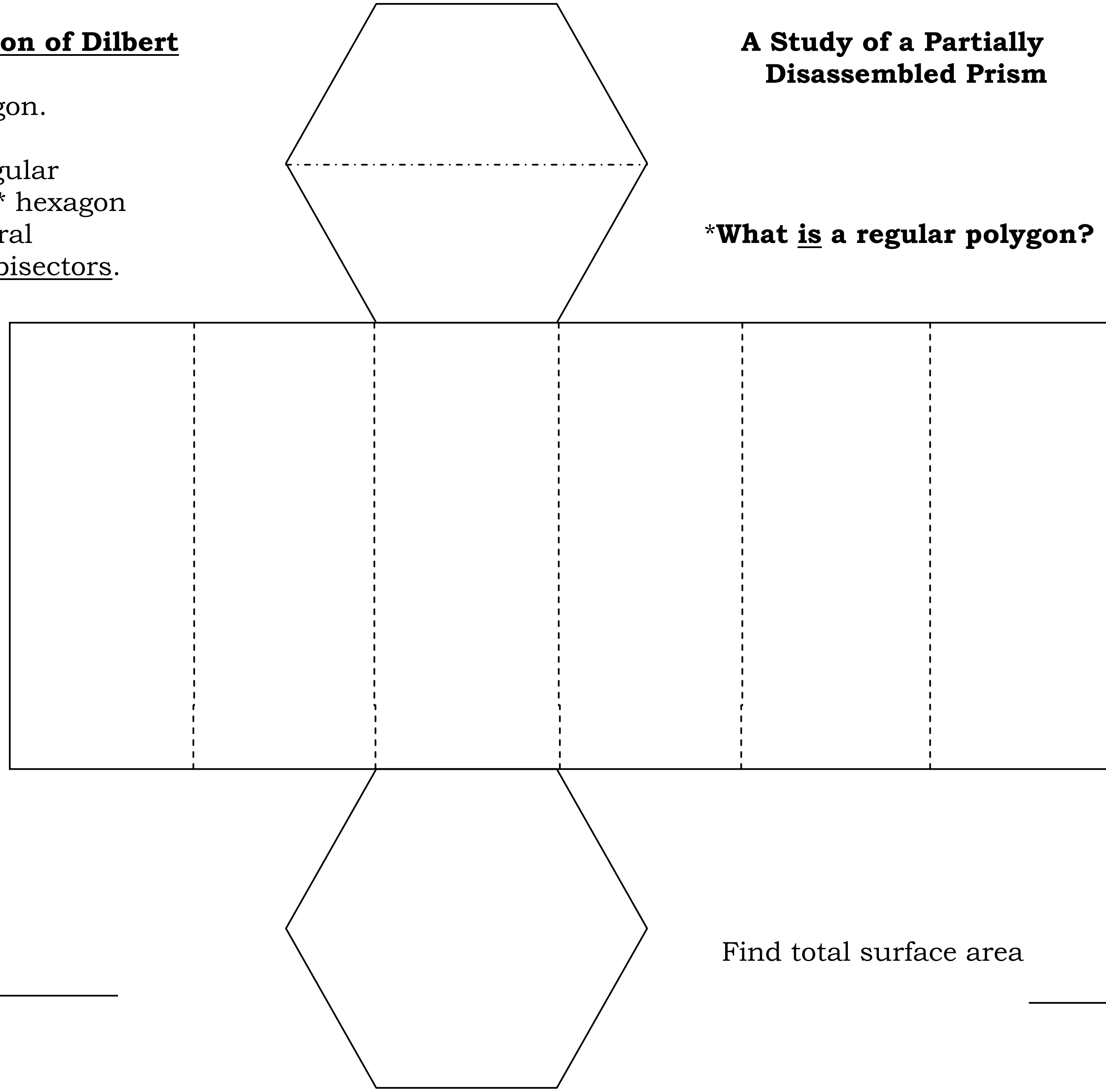
Here, $s = 3$.

Triangle area = ________

Hexagon area is _____ times as much.

Each base area = ________

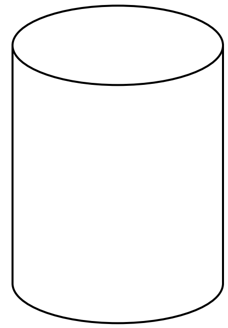
The height of this hexagonal prism is 7 cm.



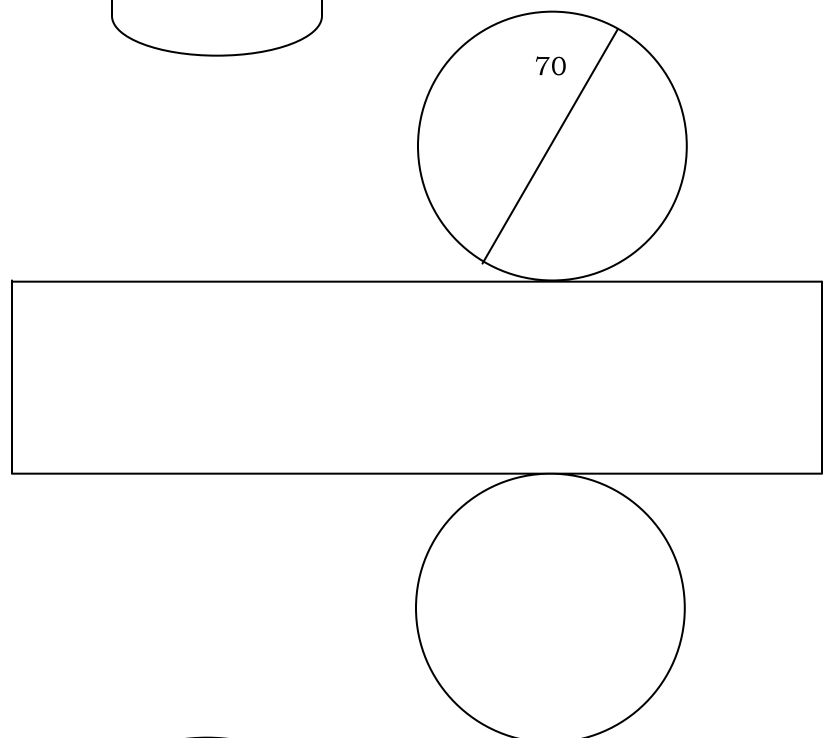
Find lateral surface area ___________

Find total surface area ____________

<u>Dilbert Reunion</u>:
A Study of a
Partially Disassembled ________________________ ,
the height of which is 50, and face diameter 70.



1. Show and complete the circle attribute schema.

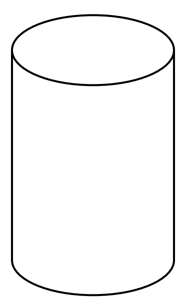


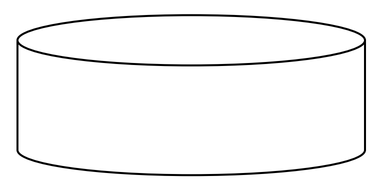
2. What is the shape of the lateral surface?

3. What is the lateral surface area of this solid figure?

4. What is the total surface area?

5. What is the volume?





Cevians and Construction Lines/Rays/Segments (CLRS) in Triangles

A *cevian* is a line or segment (a) contains the vertex of a triangle (or tetrahedron) and (b) intersects the opposite side (or face). A median is a cevian. An angle bisector is a cevian since it is a ray with the vertex as the end point. An altitude is often a cevian, but two altitudes of an obtuse triangle are not. Perpendicular bisectors are cevians only when they are also altitudes.

There is no general name for a triangle's four **C**onstruction **L**ines/**R**ays/**S**egments listed below, nor for the intersections. Proposed: *celers*, for CLRS, for construction line/ray/segment, and *celers intersections*.

"C'LERS"	***"C'LERS"*** **Intersection Point**	**Significance**
Angle bisector (ray)	Incenter	center of inscribed triangle
Perpendicular bisector (line)	Circumcenter	center of circumscribed circle
Median* (segment)	Centroid	center of mass; centroid divides each medians into parts with 2:1 ratio
Altitude** (segment)	Orthocenter	is vertex for three other triangles that replicate orthocenter concept***

* Medians are segments connecting vertices to midpoints of opposite sides. Each median cuts the triangle into two triangles of equal areas. All three medians of a triangle form six triangles of equal area.

** Together, the three altitudes of an acute triangle divide the triangle into six triangles, which are similar to each other in pairs by virtue of vertical angles.

***If **P** is the orthocenter for ΔABC, then **A** is the orthocenter for ΔPBC, **B** is the orthocenter for ΔAPC, and **C** is the orthocenter for ΔABP.

The circumcenter, orthocenter, and centroid all lay on the Euler line.

If a triangle's vertices are a,b , c,d , and e,f , the centroid coordinates are $\left(\frac{a+c+e}{3},\frac{b+d+f}{3}\right)$.

Radius r of inscribed circle, given triangle sides a, b, and c: $s = semiperimeter$; $r=\sqrt{\frac{s-a \quad s-b \quad s-c}{s}}$

Radius R of circumscribed circle, given any side of a triangle a and the opposite angle A: $R=\frac{a}{2\sin A}$

Statement Classification and Truth Tables

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Simple statements:

p: You made good grades.
q: Your parents bought your Porsche.

Negations: ("It is false that…")

$\sim p$: You didn't make good grades.
$\sim q$: Your parents didn't buy your Porsche.

Compound Statements

Conjunction: $p \wedge q$ You made grades and your parents bought your Porsche.
Disjunction: $p \vee q$ You made grades or your parents bought your Porsche.

Conditional Statements:

Implication: $p \rightarrow q$ If you make grades, then parents will buy your Porsche.
Double implication: $p \leftrightarrow q$ You make grades if and only if your parents buy your Porsche.

Truth Values

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$p \leftrightarrow q$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	F	F	T	T*	F
F	F	T	T	F	F	T**	T

* You didn't make good grades, but your parents bought you the car anyway. The rule is not broken.
** Again, the rule is not broken.