Arithmetic Catalysts for Algebra Word Problems

Dr. Stan Hartzler Archer City High School

Algebra word problems are rarely approached from the standpoint of arithmetic. For example, students are thrown into these types of word problems with almost no preview of the underlying arithmetic situations:

Consecutive integers	Angles	Coin	Chemical mixture
Distance-rate-time	Rate-of-work	Digit	Per cent mixture

An arithmetic approach may be helpful. An arithmetic background might begin with discussions and problems such as the following in the assignments several days prior to the standard problems:

For *consecutive integers*:

A. Integers are elements of the set {..., –3, –2, –1, 0, 1, 2, 3, ...} Three consecutive integers are 19, 20, 21. Four consecutive *odd* integers are 33, 35, 37, 39.

- Write the five consecutive *even* integers that precede 43.
- Represent four consecutive integers if the smallest is a.
- Represent three consecutive odd integers if the smallest is m.
- Represent five consecutive even integers if the smallest is p.

For *angle problems*:

B. Two complementary angles have sum of 90°; two supplementary angles have sum 180°. Find the measure of an angle that is complementary to a 34° angle; then find the supplement of the same angle. Similar exploration for angle sums in triangles should precede related word problems.

- An angle has measure of $x°$. Represent its complement.
- An angle has measure of $y°$. Represent its supplement.
- Two angles of a triangle have measure of $z°$. Represent the measure of the third angle.

For *distance-rate-time*:

C. How far does a plane travel in three and one-half hours at a rate of 460 miles per hour? How long does it take a cyclist to go 91 miles at 13 miles per hour? What is the rate of a swimmer who does $3a$ laps in $5x^2$ minutes?

For *rate-of-work*:

D. How much of a job gets done if someone does 1/8 of the job each hour and works for 5 hours?

E. What fraction of a job gets done if a person can finish the job in 20 minutes but only works for 12 minutes?

F. What fraction of a job gets done if two people work on it together for 20 minutes, one at a rate of 1/60 of the job per minute and the other at a rate of 1/45th of the job per minute?

G. What is the rate of work, per minute, for a machine that can finish a job in 13 hours?

H. What fraction of a job gets done if three people work on it for 15 minutes, the first able to finish alone in 60 minutes, the second in 80 minutes, and the third in 100 minutes?

H′. What fraction of a job gets done if three people work on it for b minutes, the first able to finish alone in x minutes, the second in y minutes, and the third in z minutes?

For <u>*digit*</u>:

I. If a number has 5 for a ten's digit and 8 for a one's digit, what is the number represented by the digits?

J. What arithmetic do we do to these digits to get the number?

K. What is the number with the digits reversed?

L. What is the difference between the original number and the reversed number?

M. What number is made if the ten's digit is x and the one's digit is y? Then what if these digits are reversed?

For <u>*per cent mixture*</u>:

N. If 20 pounds of beach sand contains 2 pounds of tar, what per cent of the 20-pound mix is tar?

O. If five pounds of pure sand is added, what per cent is tar?

P. If five pounds of pure tar is added to the original mix, what per cent is now tar?

A discussion of relevance is appropriate here, as many in mathematics education have suggested that these problem types are contrived and useless to students. Others have answered that each type described above seems to lend itself to learning some kinds of skills.

The latter position will be supported here. For example:

- The parallel-opposing forces situation in distance-rate-time problems gives students practice in setting up systems of equations to solve word problems, as well as exemplifying physics.
- Coin problems help students learn to distinguish relationships between quantities of represented values (numbers of certain coins) from relationships in the actual values (monetary worth of coins).
- Digit problems set a stage for study of properties of numbers, such as the principles behind casting out nine's, in number theory.