Quadratic Formula Development and Graph

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When you can't factor a _____ _____ like $y = f(x) = 2x^2 + 3x + 2 = 0$,

(A) complete the square, *which no one does outside of courses.* **(B)** graph; look for _____

(C) use the quadratic formula: Given $y = f(x) = ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

steps	$y = f(x) = 2x^2 + 3x + 2 = 0$	$y = f(x) = ax^2 + bx + c = 0$
1. Isolate the terms containing x on the left, with spaces. Non-x constant number must be alone on the right.	1	
2. If coefficient of x^2 is not one, divide each term by coefficient of x^2.	2,4,5	
3. Set up completion of square structure: parentheses and exponent, with x, + or −, and half of x coefficient		
4. Complete square above structure with (half of x coefficient)^2, added to both sides.	3,5	
5. Add the terms on the right side.	6	
6. Write the square root of both sides, using a $\pm$ symbol in front of the radical sign on the right side.	7	
7. Solve for x.		

1. Solutions/roots/zeroes/answers:

If $a > 0$, $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

2. Axis of symmetry: $x = \frac{-b}{2a}$

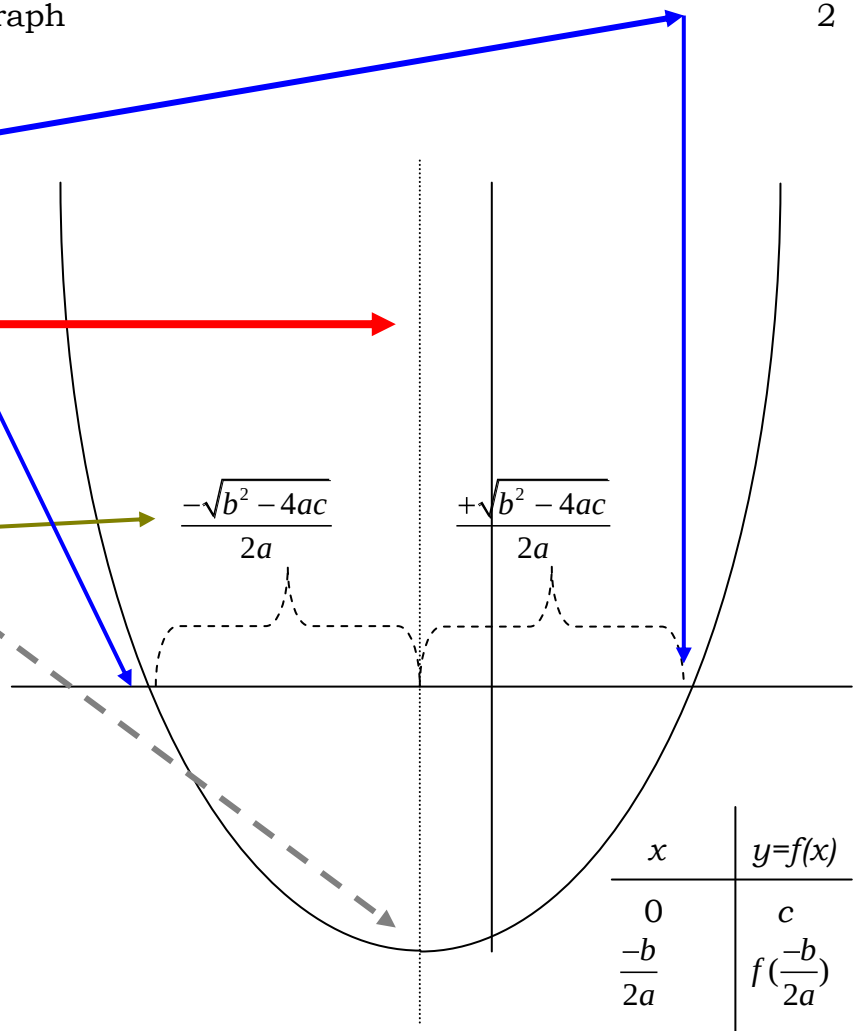
3. Vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

4. Discriminant: $\sqrt{b^2 - 4ac}$

- If **Discriminant = 0**, then the "two" solutions are equal to each other – **ONE solution** in reality. The "two" intersections with the *x* axis are ONE in reality and are the vertex.
- If **Discriminant < 0**, then the "two" solutions are imaginary – **NO real solutions**. There are no intersections with the *x* axis.
- If **Discriminant > 0**, then there really are **TWO unequal real** solutions, and they differ because of the ± choice. The $x = \frac{-b}{2a}$ symmetry value is an average of the two roots therefore.

Notes:

- If $a < 0$, the whole parabola gets turned upside down.



x	$y=f(x)$
0	c
$\frac{-b}{2a}$	$f\left(\frac{-b}{2a}\right)$
$\frac{-b + \sqrt{b^2 - 4ac}}{2a}$	0
$\frac{-b - \sqrt{b^2 - 4ac}}{2a}$	0

$$ax^2 + bx + c = 0$$

$$ax^2 + bx + \quad = -c$$

$$\frac{ax^2}{a} + \frac{bx}{a} + \quad = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}\frac{\bullet 4a}{\bullet 4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$-\frac{b}{2a} = -\frac{b}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For a quiz or exam, one may omit the second line and still get full credit.