Odd and Even Functions Extravaganza

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	ODD	**EVEN**
	$y = f(x) = 3x^5 - 8x^3 + 3x$	$y = g(x) = x^4 - 8x^2 + 9$
1. What about these functions? (exponents?)		
2. Graph the functions.		
3A. What do you notice about the graphs? (*Symmetry*?) 3B. The wire discussions?		
4. What is awesome about segments from a point on the graph?		
5. Complete charts. Pattern? Tie to graphs? *Note graph point pairs.* Substitute (by hand): $f(2)$, $f(-2)$, g(2), g(−2) $f(2) = 3 \bullet 2^5 - 8 \bullet 2^3 + 3 \bullet 2$ $f(-2) = 3(-2)^5 - 8(-2)^3 + 3(-2)$ $g(2) = 2^4 - 8(2)^2 + 9$ $g(-2) = (-2)^4 - 8(-2)^2 + 9$ Observation? Why?		
6A. Summarize odd/ even functions per parent-function transformations. 6B. Analyze.		
7. If x, y is on graph…		

Odd and Even Functions Extravaganza KEY?

<table>
<tr><th></th><th>ODD</th><th>EVEN</th></tr>
<tr><td></td><td>$y = f(x) = 3x^5 - 8x^3 + 3x$</td><td>$y = g(x) = x^4 - 8x^2 + 9$</td></tr>
<tr><td>1. What about these functions? (exponents?)</td><td>All exponents are odd.</td><td>All exponents are even.</td></tr>
<tr><td>2. Graph the functions.</td><td></td><td></td></tr>
<tr><td>3A. What do you notice about the graphs? (Symmetry?)

3B. The wire discussions?</td><td><u>Symmetrical with respect to the origin.</u>
If graph for $x > 0$ were a bent wire, a 180° rotation about (0,0) would place wire atop graph for $x < 0$.</td><td><u>Symmetrical with respect to the y axis.</u>
If graph for $x > 0$ were a bent wire, a flip across $x = 0$ (y-axis) would place wire atop graph for $x < 0$.</td></tr>
<tr><td>4. What is awesome about segments from a point on the graph?</td><td>Segment from a graph point to origin has twin on other side.</td><td>Segment from a graph point to y axis ($\perp$) has twin on other side.</td></tr>
<tr><td>5. Complete charts.
Pattern? Tie to graphs?
Note graph point pairs.
Substitute (by hand):
$f(2)$, $f(-2)$, g(2), g(−2)
$f(2) = 3 \bullet 2^5 - 8 \bullet 2^3 + 3 \bullet 2$
$f(-2) = 3(-2)^5 - 8(-2)^3 + 3(-2)$
$g(2) = 2^4 - 8(2)^2 + 9$
$g(-2) = (-2)^4 - 8(-2)^2 + 9$
Observation? Why?</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>−3</td><td>$f(-3) = -522$</td></tr><tr><td>−2</td><td>$f(-2) = -38$</td></tr><tr><td>−1</td><td>$f(-1) = 2$</td></tr><tr><td>0</td><td>$f(0) = 0$</td></tr><tr><td>1</td><td>$f(1) = -2$</td></tr><tr><td>2</td><td>$f(2) = 38$</td></tr><tr><td>3</td><td>$f(3) = 522$</td></tr></table>Odd exponents change no input signs. $f(3)$ is opposite of $f(-3)$.

Note graph point pairs.</td><td><table><tr><th>x</th><th>y</th></tr><tr><td>−3</td><td>18</td></tr><tr><td>−2</td><td>−7</td></tr><tr><td>−1</td><td>2</td></tr><tr><td>0</td><td>9</td></tr><tr><td>1</td><td>2</td></tr><tr><td>2</td><td>−7</td></tr><tr><td>3</td><td>18</td></tr></table>Even exponents turn negative input to positive results.
$f(-3) = f(3)$.
Note graph point pairs.</td></tr>
<tr><td>6A. Summarize odd/even functions per parent-function transformations.

6B. Analyze.</td><td>$f(x) = -f(-x)$
<u>x axis flip + y axis flip</u>
means no change in graph of $f(x)$.</td><td>$f(x) = f(-x)$
<u>y axis flip</u>
means no change in graph of $f(x)$.</td></tr>
<tr><td>7. If (x,y) is on graph…</td><td>…then ($-x$, $-y$) is, too.</td><td>…then ($-x$, y) is, too.</td></tr>
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