Pythagorean Triples per Sierpinski page 8 and Proof of Right Triangle Area = *xy*

"Every primitive Pythagorean triple (a, b, c) where b is an even integer is obtained only once per the following, where u and v are odd and $u > v$:

$$a = uv \qquad b = \frac{u^2 - v^2}{2} \qquad c = \frac{u^2 + v^2}{2}$$

Here, u and v represent all pairs of odd, relatively prime natural numbers." -- Waclaw Sierpinski, *Pythagorean Triangles, The Scripta Mathematics Studies Number Nine.* New York City: Graduate School of Science, Yeshiva University, 1962.

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Proof of Right Triangle Area = *xy*

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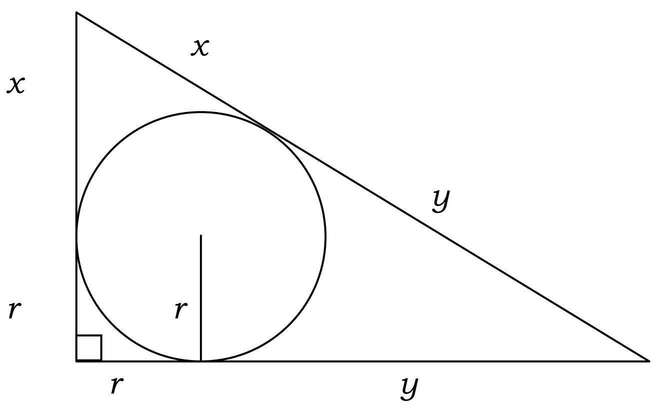

Area = $\frac{1}{2} r \bullet$ perimeter = $\frac{1}{2} r(2r + 2x + 2y) = r(r + x + y)$

$$(x + r)^2 + (y + r)^2 = (x + y)^2$$
$$x^2 + 2xr + r^2 + y^2 + 2yr + r^2 = x^2 + 2xy + y^2$$
$$2xr + r^2 + 2yr + r^2 = 2xy$$
$$2xr + 2r^2 + 2yr = 2xy$$
$$xr + r^2 + yr = xy$$
$$r(x + r + y) = xy = \text{ AREA}$$

Fun Relationships

$$r = \frac{a+b-c}{2} = \frac{v(u-v)}{2} = \frac{\sqrt{x^2+6xy+y^2}-(x+y)}{2}$$

$$x = a - r = \frac{v^2+uv}{2}$$
$$y = b - r = \frac{u^2-uv}{2}$$

$$a = x + r = \frac{\sqrt{x^2+6xy+y^2}+x-y}{2}$$
$$b = y + r = \frac{\sqrt{x^2+6xy+y^2}-x+y}{2}$$
$$c = x + y$$

Perimeter = $a+b+c = 2(r+x+y) = x+y+\sqrt{x^2+6xy+y^2} = uv+u^2$

Area = $\frac{ab}{2} = xy = \frac{u^3v-uv^3}{4}$

Developing *u*, *v* for a given *r*

Example: $r = 3 \times 5 \times 7 = 105$

$$r = \frac{uv-v^2}{2} \Rightarrow u = \frac{v^2+2r}{v} = \frac{v^2+210}{v}$$

v	*u*	*a*	*b*	*c*
1	211	211	22260	22261
3	73	219	2660	2669
5	47	235	1092	1117
7	37	259	660	709
15	29	435	308	533
21	31	651	260	701
35	41	1435	228	1453
105	107	11235	212	11237

The number of distinct values of (u, v) for a given r of n odd prime factors is 2^n. And note that $u_k + v_k = u_{2^n+1-k} + v_{2^n+1-k}$.