Deductive Structure in Geometry and…

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Veterans of decent geometry courses will recall some of the following elements, and perhaps some of the structure as well.

Undefined Terms:
- point
- line
- plane
- space … between (?)…

Definitions (from undefined terms):
- ray, segment, polygon,…
- parallel, perpendicular, skew,…
- acute, right, obtuse…
- scalene, isosceles, equilateral,…

Unproved Hypotheses (built on experience with undefined and defined ideas):
- Parallel Postulate
- SSS, SAS, ASA $\Delta \cong$
- reflexive, symmetric, transitive
- corresponding angles are $\cong$.
- AA $\Delta \sim$
- … and *uncountably* many more.

Theorems (built from terms, hypotheses, and other theorems):
- Vertical angles are congruent
- All right angles are congruent.
- AAS $\Delta \cong$
- Pythagorean Theorem
- Alternate interior angles are $\cong$.
- $\sum$ of Δ $\angle$ measures = 180°
- … and *uncountably* many more.

In geometry, attention is given to the notion that *all postulates would dearly love to become theorems*. There is a weird status system working here, even stranger that the status systems that exist among people.

The effort to turn the Parallel Postulate into a theorem led to other kinds of geometry, the *non-Euclidean geometries*, discussed briefly in ACISD geometry classes.

Attention should be given to the issue of the *uncountable* collection of Euclidean geometry postulates. In an effort to gain some strange status for humanity, many mathematicians have tried to show that mathematics is entirely a human invention.

These efforts failed. A 20th-Century German mathematician, Gödel, showed that for any mathematics system, including arithmetic, complete knowledge cannot be obtained deductively with a countable list of assumptions (postulates). Gödel's proof of this idea is rather simple, and gave him rock-star-plus status among great people of the 20th Century.

The countable issue is a fun issue. The set of **rational numbers**, while infinite, is a countably infinite set. The set of **irrational numbers** is uncountably infinite. So is the set of geometry postulates.

For most mathematicians, this shoots down the idea that truth in mathematics comes entirely from people. This raises a question: *"Then where does it come from?"* Some status-seekers are very troubled by this question. Your teacher is not troubled at all -- in fact, delighted -- by that question. DONE.