Cevians and Construction Lines/Rays/Segments (CLRS) in Triangles

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A *cevian* is a line or segment (a) contains the vertex of a triangle (or tetrahedron) and (b) intersects the opposite side (or face). A median is a cevian. An angle bisector is a cevian since it is a ray with the vertex as the end point. An altitude is often a cevian, but two altitudes of an obtuse triangle are not. Perpendicular bisectors are cevians only when they are also altitudes.

There is no general name for a triangle's four **C**onstruction **L**ines/**R**ays/**S**egments listed below, nor for the intersections. Proposed: *celers*, for CLRS, for construction line/ray/segment, and *celers intersections*.

"C'LERS"	***"C'LERS"*** **Intersection Point**	**Significance**
Angle bisector (ray)	Incenter	center of inscribed triangle
Perpendicular bisector (line)	Circumcenter	center of circumscribed circle
Median* (segment)	Centroid	center of mass; centroid divides each medians into parts with 2:1 ratio
Altitude** (segment)	Orthocenter	is vertex for three other triangles that replicate orthocenter concept***

* Medians are segments connecting vertices to midpoints of opposite sides. Each median cuts the triangle into two triangles of equal areas. All three medians of a triangle form six triangles of equal area.

** Together, the three altitudes of an acute triangle divide the triangle into six triangles, which are similar to each other in pairs by virtue of vertical angles.

***If **P** is the orthocenter for ΔABC, then **A** is the orthocenter for ΔPBC, **B** is the orthocenter for ΔAPC, and **C** is the orthocenter for ΔABP.

The circumcenter, orthocenter, and centroid all lay on the Euler line.

If a triangle's vertices are a,b , c,d , and e,f , the centroid coordinates are $\left(\frac{a+c+e}{3},\frac{b+d+f}{3}\right)$.

Radius r of inscribed circle, given triangle sides a, b, and c: $s = semiperimeter$; $r=\sqrt{\frac{s-a\quad s-b\quad s-c}{s}}$

Radius R of circumscribed circle, given any side of a triangle a and the opposite angle A: $R=\frac{a}{2\sin A}$