Conics "Constructed" by "Manipulatives"

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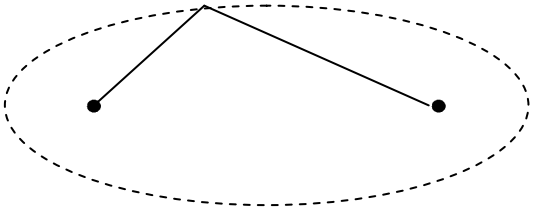
A. Circle: Use compass.

B. Ellipse:

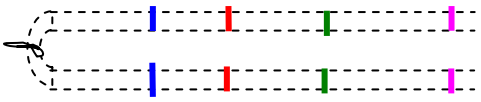
1. Anchor two nails or hooks on a flat surface. There are fixed points. Distance between is $2c$.
2. Tie ends of string to hooks. Length of string between hooks is 2a.
3. Tighten string with tip of pencil, keeping string flat on surface.
4. Move string per directions above as far as possible in either direction, on both sides of the segment joining the fixed points.

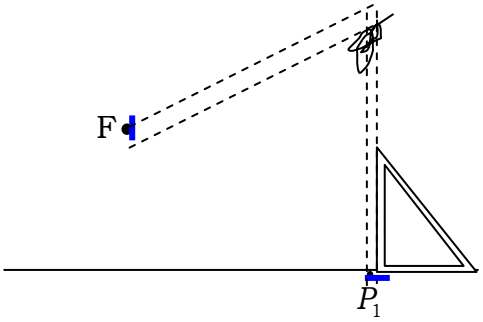


C. Parabola:

Materials: string, marker, and triangle or other drafting device for drawing perpendiculars.

1. Draw a point F (focus) and a line (directrix) on a surface.
2. Prepare a string with a knot in or near the middle, and markings at equal distances from the knots.





3. Place one blue mark on the focus point.
4. Place triangle on line as shown, so that
   - the second blue mark is on the line,
   - the string is perpendicular to the line (per triangle), and
   - the knot is extended fully away from both blue marks.
5. Mark parabola point where knot is.
6. Repeat for other marks.

"Construction" option: 1. Draw FP_1 and perpendicularly bisect it.

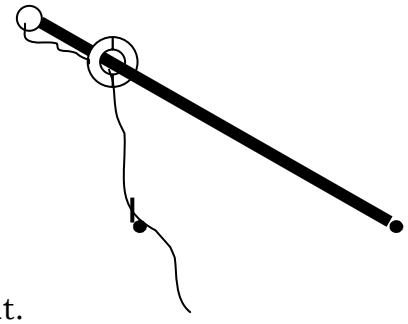
2. Move and adjust compass so that compass point Q_1 is on $\perp$ bisector and distance from to F equals distance from Q_1 to P_1. Mark Q_1.

3. Choose another P_2 and find another Q_2

D. Hyperbola

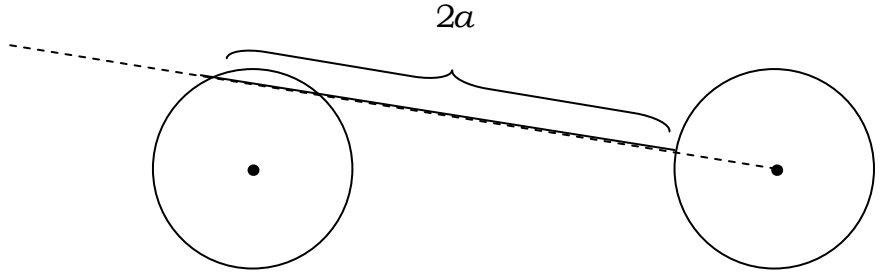
Materials: Stick (18-24 inches long) with anchor (eyelet?) at end, string that is length of stick, washer (or rubber band).

Assemble string, stick, and washer (or rubber band)as shown:



1. Mark string at a point that is $2a$ from the end of the string.
2. Draw two fixed points (focus points) on a flat surface. Distance between is $2c$, where $c > a$.
3. Anchor string mark to one focus point. Drawing has the left point as the string anchor.
4. Hold end of stick at other focus point.
5. Tighten string with washer or by stick rotation, keeping string flat on surface.
6. Mark washer location. This is a point on hyperbola.
7. Move washer and rotate stick to new locations.

"Construction" option: 1. Draw equal circles centered at focus points.
2. Place ruler so that edge is at right focus point and ruler mark at its circle is clear.
3. Measure a $2a$ length from circle to other (left-side) circle, adjusting ruler as needed.
4. When $2a$ length has been found from right circle to left circle, mark point on left circle. This point is a hyperbola point.
5. Repeat for circles of other radii.



To construct auxiliary rectangle,
(1) Construct line joining focus points, and its perpendicular bisector.
(2) Construct right triangle with leg a and hypotenuse c. Other leg = b.
(3) Construct two segments of length $2b$ $\perp$ to segment joining focus points, at each vertex and with each vertex the midpoint. Ends of these two segments determine auxiliary rectangle.