Permutation and Combination
Concept Development

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This writer first encountered permutations and combinations in an upper-division probability and statistics course in 1967, and then spent ten more years getting the ideas straight. Students in middle grades classes, by contrast, are now expected to make the distinction and count the possibilities. Confusion expressed by one of the teachers at a training workshop in summer 1999 motivates the formal discussion that follows, based on an organization used by in-service and pre-service audiences to help develop a conceptual understanding.

Definitions: A *permutation* is an ordered or arranged subset; a *combination* is an unordered subset. (The reader is expected to get nothing from those definitions at this point.)

Books:

Permutation. The boss is coming over for dinner and an impressive bookshelf is needed. Ten books of varying colors, sizes, and topics are available, with room on the shelf for six. *Different* arrangements (orders) of each subset of six will look different -- the heights might decrease from left to right, or increase; perhaps the tallest should be in the middle, and so on. How many six-book arrangements (*permutations*) are possible?

Combination. My wife has a list of ten book titles that she would like for Christmas. I will buy six of these books, and put the books in a box. Each gift-collection of six books (*combination*) will be the *same* for her no matter what order she takes them from the box. How many gifts are possible?

(The reader should still not expect to understand anything yet.)

Officers in a Club

Permutation. The mathematics club has ten members. The *different* offices are President, Vice-President, and Bouncer. If Alicia is President, Bill is Vice President, and Chuck is Bouncer, we have a different slate of officers from that if Bill is President, Alicia is VP, and Chuck is Bouncer. How many arrangements (*permutations*) are possible if all ten members are eligible?

Combination. The mathematics club has a cleanup committee consisting of Alice, Bill, and Chuck, all with the *same* committee membership. How many such cleanup committees (*combinations*) are possible if all ten members are eligible?

(The reader might begin to understand at this point, but if not, just keep on reading.)

Alphabet Letters

Permutation. The word *was* is different from the word *saw* because of the arrangement (order) of letters. How many different words (*permutations*) are possible if each letter of the alphabet may be used once at most?

Combination. A Wheel-of-Fortune contestant spins the wheel; the marker stops at a spot that says, "Pick three letters." The order that the contestant calls them out is not important; Vanna will still turn them all before the game proceeds. How many three-letter guesses (*combinations*) are possible from the eligible alphabet letters? Alternative: differing letters distributed in a Scrabble game.

(The reader should be catching on. The summary chart at the end might be helpful.)

Prizes

Permutation. The first-place winner gets $500; second-place gets a *different* $300; third-place gets a *different* $100. The winnings differ if the first-place and second-place winners trade places. How many ordered sets of place winners (*permutations)* are possible from 50 entries in the raffle?

Combinations. The three top winners in a raffle each get the *same* $400; the order doesn't matter. How many sets of top winners (*combinations)* are possible from 50 entries?

Digits

Permutation. The number 452 is *different* from the number 524 because of the different order. How many different three-digit numbers (*permutations*) are possible if the digits are chosen from 1 through 9?

Combination. In poker and other card games, the order that cards are dealt to each player doesn't matter; the player arranges and plays cards without any attention to the order in which the cards were received. If a player is dealt a four, then a five, and then a two, this is the *same* as being dealt a five, then a two, then a four. How many different poker hands (*combinations*) are possible if each hand consists of five cards dealt from a deck of 52 cards?

SUMMARY CHART

Model	**Permutation**	**Combination**
Books	on a shelf	in a gift box
Officers	distinct offices	committee
Letters	in a word	in a set
Prizes	distinct ranking	top tier
Digits	in a number	in a hand of cards

Quiz: Thirty-three ice cream flavors are available. Three are to be chosen for either a cone or a smoothie. (a) Is the cone a permutation or a combination? (b) How about the smoothie? (c) Does order matter to children in a lunch line? (d) Does order matter to children grouped about a piñata?

Mathematics Language and Details
(High school level?)

	Permutation	**Combination**
notation	$(3, -1) \neq (-1,3)$	$\{3, -1\} = \{-1,3\}$
abbreviation	${}_nP_r$	${}_nC_r$
idea	"n things <u>arranged</u> r at a time"	"n things <u>chosen</u> r at a time"
formula*	${}_nP_r = \dfrac{n!}{(n-r)!}$	${}_nC_r = \dfrac{n!}{(n-r)!r!}$

*The distinction between the two formulas is this: in the Combinations denominator, we divide out the number of rearrangements that are possible for each set, that number being $r!$.

Permutation specialties

Letters that repeat in a word (such as *Mississippi*): divide out repeaters.

Example: *Mississippi* has 11 letters, with i and s appearing four times each, and p appearing two times. The number of permutations using all 11 letters is

$$\frac{11!}{4!\,4!\,2!}$$

If n people sit in a circle, the number of permutations = $(n-1)!$

If n keys are arranged in a ring, the number of permutations is $\dfrac{(n-1)!}{2}$.

Three Essential Basics for Radicals

1. $\sqrt{3} \bullet \sqrt{3} = 3$ 2. $\sqrt{3} \bullet \sqrt{2} = \sqrt{6}$ 3. $\sqrt{5} + \sqrt{5} + \sqrt{5} = 3\sqrt{5}$

Students can verify these, and check/disprove propositions such as

"Does $\sqrt{3} + \sqrt{2} = \sqrt{5}$?"