Distributivity and <u>*NON*</u>–Distributivity in Algebra

GOOD:

$2(x+3y)=2x+6y$	multiplication	**O**	addition
$4(w-3z)=4w-12z$	or	**V**	or
$\frac{1}{3}(6a+b)=2a+\frac{b}{3}$	division	**E**	subtraction
$\left(2x^2y^4\right)^3=8x^6y^{12}$	exponentiation	**R**	multiplication

WRONG: $(x + z)^2 = ?$ **<u>NO</u>:** x^2+z^2 is **<u>NOT OK</u>**

Operation	Distributivity of Exponentiation	Distributivity of Square Root
Multiplication	$(3\bullet 5)^2=225=3^2\bullet 5^2$	$\sqrt{4\bullet 9}=\sqrt{4}\bullet\sqrt{9}$
Division	$\left(\frac{3}{5}\right)^2=\frac{3}{5}\bullet\frac{3}{5}=\frac{3^2}{5^2}$	$\sqrt{36\div 9}=\sqrt{36}\div\sqrt{9}$

Operation	<u>NON</u>–Distributivity of Exponentiation	<u>NON</u>–Distributivity of Square Root
Addition	$(x+y)^2=x^2+2xy+y^2$ $\neq x^2+y^2$	$\sqrt{4}+\sqrt{9}\neq\sqrt{4+9}$
Subtraction	$(x-y)^2=x^2-2xy+y^2$ $\neq x^2-y^2$	$\sqrt{36}-\sqrt{9}\neq\sqrt{36-9}$

<u>Addition with *equal addends* is multiplication</u>, and **multiplication distributes over addition**.

<u>Multiplication with *equal factors* is exponentiation</u>, and **exponentiation distributes over multiplication**.

"Addition" above may be replaced with "subtraction."
"Multiplication" above may be replaced with "division."
"Exponentiation" above extends to real-number exponents.

So square roots, cube roots, etc.. distribute over multiplication and division, but <u>not</u> over addition and subtraction.

Main Idea:

Distributivity is *not* permitted whenever the notion just occurs to the human mind.

Next Main Idea:

Distributivity sometimes makes interesting (unexpected) changes when it ***is*** permitted…

EXOTIC (Unexpected) DISTRIBUTIVITY

DeMorgan's Laws for Sets and Logic

"The complement ~ of the union $\cup$ of sets A and B equals the ***intersection*** $\cap$ of the *separate* complements ~A and ~B"

In Symbols: $\sim(A \cup B) = (\sim A) \cap (\sim B)$

Also, $\sim(A \cap B) = (\sim A) \cup (\sim B)$

U N E X P E C T E D ! U N E X P E C T E D ! U N E X P E C T E D !

The distributivity switches the binary operations union and intersection.

The same laws apply in logic, where

- instead of complement, negation is the unary operation,
- instead of sets, statements are the elements
- instead of "union" and "intersection", "or" and "and" are the binary operations.

DeMoivre's Thorem

With a complex number written in polar form, $r \operatorname{cis} \theta$,

$$(r \operatorname{cis} \theta)^n = r^n cis(n\theta)$$

Note that the exponent n distributes to r as an exponent, but to θ as a coefficient.

U N E X P E C T E D ! U N E X P E C T E D ! U N E X P E C T E D !