But it looks easy –

$7+0=7$ $7+1=8$

$7\times0=0$ $7\times1=7$

$7^0=1$ $7^1=7$

– and it <u>isn't</u> <u>easy</u>.

Distributivity and ***NON***–Distributivity in Algebra

GOOD:

$2\ \ x+3y\ = 2x+6y$	multiplication		addition
$4\ \ w-3z\ = 4w-12z$	or	**O**	or
$\frac{1}{3}\ \ 6a+b\ = 2a+\frac{b}{3}$	division	**V E R**	subtraction
$2x^2y^4\ ^3 = 8x^6y^{12}$	exponentiation		multiplication

WRONG: $x\ +\ z\ ^2 = ?$ **NO:** x^2+z^2 is **NOT OK**

Operation	**Distributivity of Exponentiation**	**Distributivity of Square Root**
Multiplication	$3\bullet 5\ ^2 = 225 = 3^2\bullet 5^2$	$\sqrt{4\bullet 9} = \sqrt{4}\bullet\sqrt{9}$
Division	$\left(\frac{3}{5}\right)^2 = \frac{3}{5}\bullet\frac{3}{5} = \frac{3^2}{5^2}$	$\sqrt{36\div 9} = \sqrt{36}\div\sqrt{9}$

Operation	**NON–Distributivity of Exponentiation**	**NON–Distributivity of Square Root**
Addition	$x+y\ ^2 = x^2+2xy+y^2$ $\neq x^2+y^2$	$\sqrt{4}+\sqrt{9}\neq\sqrt{4+9}$
Subtraction	$x-y\ ^2 = x^2-2xy+y^2$ $\neq x^2-y^2$	$\sqrt{36}-\sqrt{9}\neq\sqrt{36-9}$

Addition with *equal addends* is multiplication, and **multiplication distributes over addition**.

Multiplication with *equal factors* is exponentiation, and **exponentiation distributes over multiplication**.

"Addition" above may be replaced with "subtraction."
"Multiplication" above may be replaced with "division."
"Exponentiation" above extends to real-number exponents.

So square roots, cube roots, etc.. distribute over multiplication and division, but not over addition and subtraction.

Main Idea:

Distributivity is *not* permitted whenever the notion just occurs to the human mind.

Next Main Idea:

Distributivity sometimes makes interesting (unexpected) changes when it ***is*** permitted…

EXOTIC (Unexpected) DISTRIBUTIVITY

DeMorgan's Laws for Sets and Logic

"The complement ~ of the union $\bigcup$ of sets A and B equals the ***intersection*** $\bigcap$ of the *separate* complements ~A and ~B"

In Symbols: $\sim A \bigcup B = \sim A \bigcap \sim B$

Also, $\sim A \bigcap B = \sim A \bigcup \sim B$

U N E X P E C T E D ! U N E X P E C T E D ! U N E X P E C T E D !

The distributivity switches the binary operations union and intersection.

The same laws apply in logic, where

- instead of complement, negation is the unary operation,
- instead of sets, statements are the elements
- instead of "union" and "intersection", "or" and "and" are the binary operations.

DeMoivre's Thorem

With a complex number written in polar form, $r\,\text{cis}\,\theta$,

$$r\,\text{cis}\,\theta^{\;n} = r^n cis\;\; n\theta$$

Note that the exponent n distributes to r as an exponent, but to θ as a coefficient.

U N E X P E C T E D ! U N E X P E C T E D ! U N E X P E C T E D !

Changing Greater-Than for Less-Than and Vice Versa

Four solving situations require changing greater-than “>” for less-than “<”.

All four originate with arithmetic.

1. **REVERSE:**

When a solution appears in an order contrary to the customary left-to-right order.

Solution:
$$-6 < 2x$$
$$-3 < x$$

More *readable*: $x > -3$

2. **NEGATIVE FACTOR:**

When multiplying or dividing two or three members of an inequality by a negative number.

$$-5w > 4$$
$$w < -\frac{4}{5}$$

3. **INVERT:**

When writing the reciprocal of two members of an inequality statement. (Denominators such as z below must be positive; otherwise, complications occur.)

If $\frac{1}{z} < \frac{3}{4}$ then $z > \frac{4}{3}$

If $\frac{1}{z} > \frac{6}{5}$ then $z < \frac{5}{6}$

4. **LOG/EXPONENTIATE FORM CHANGE:**

When changing the form between logarithm equation and exponent equation, AND the base of the logarithm is between zero and one.

$$\log_{\frac{2}{3}} x < 5 \quad \Leftrightarrow \quad x > \left(\frac{2}{3}\right)^5$$

Rational ***Expressions*** vs. Rational ***Equations***

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Expressions	***Equations***
Principle: Preserve *Value*	**Principle: Preserve *Equality***
Example: "Simplify" $$\frac{x}{x+1}+\frac{1}{x-1}=?$$	Example: "Solve for *x*" $$\frac{x}{x+1}+\frac{1}{x-1}=x$$
Procedure: add over common denominator ***AND*** ***KEEP the denominator in the answer***	Procedure: multiply both sides by common multiple ***AND*** ***DESTROY the denominators***

When Do Solutions Disappear in Algebra?

1. When variables in denominators turn denominators into zero.
2. When variables in radicands turn radicands into negative numbers.
3. When a logarithmic equation produces base $b \leq 0$ for the logarithm.

Three Key Ideas for Understanding Radicals

1. $\sqrt{3} \bullet \sqrt{3} = 3$ 2. $\sqrt{3} \bullet \sqrt{2} = \sqrt{6}$ 3. $\sqrt{5} + \sqrt{5} + \sqrt{5} = 3\sqrt{5}$

Unless these three principles are learned, further study of radicals will be difficult. Practice in discriminating between these three can begin in middle grades, and should occur as part of a daily review routine.

Helpful descriptors?

1. Definition of square root 2. Arithmetic fact; verify with calculator.
3. Meaning of multiplication.

Trigonometric Function Values for "Nice" Angles

$\angle$ °	0	30	45	60	90
$\angle$ rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sine	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cos	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
tan	$\frac{\sqrt{0}}{3}$	$\frac{\sqrt{3}}{3}$	$\frac{\sqrt{9}}{3}$	$\frac{\sqrt{27}}{3}$	∞

Six Connections for Systems of Equations

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The column headings are standard descriptions of systems of equations. The left-hand column lists issues related to solving these systems.

	Consistent and Independent	**Consistent and Dependent**	**Inconsistent**
What happens when solution by linear combination or substitution is attempted:	x or y equals a number	x and y both vanish; result looks like $0 = 0$ or $-2 = -2$	x and y both vanish; result looks like $0 = -17$ or $42 = 29$
What the solution set looks like:	$(x,y) = (1,-5)$	$\{ (x,y) \mid y = x + 1 \}$	$\varnothing$
What the graph looks like:	Intersecting lines	Same line	Parallel lines
How the equations appear:	Nothing unusual	$x + y = 2$ $3x + 3y = 6$	$x + y = 2$ $x + y = 3$
What happens when Cramer's Rule is applied:	Denominator $\neq 0$	Numerator and denominator $= 0$	Denominator only $= 0$
What happens when Gauss-Jordan is applied:	Coefficient matrix is row-equivalent to identity matrix	Entire row of zeroes appears <u>including</u> the constant term	Entire row of zeroes appears <u>except</u> the constant term

Greatest Common Factor, Lowest Common Multiple

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To find Greatest Common Factor and Lowest Common Multiple of 80 and 150:

Factor Lists			
150		**80**	
1	150	1	80
2	75	2	40
3	50	4	20
5	30	5	16
6	25	8	10
10	15		
Greatest **C**ommon **F**actor: 10			

Prime Factorization	
150	80
15 10	8 10
3 5 2 5	2 2 2 2 5
$2 \bullet 3 \bullet 5^2$	$2^4 \bullet 5$
Greatest **C**ommon **F**actor: $2 \bullet 5$	

III. Introducing the "divides" bar.

The statement " 6 | 18 " means "6 divides into 18 without remainder."

To find GCF, a smaller number, write the given numbers on the right of the "divides" bars. The number in the blank must be the biggest collection of factors that will divide into both prime factorizations.	____ | $2 \bullet 3 \bullet 5^2$ | ____ | $2^4 \bullet 5$ |	To find LCM, a larger number, write the given numbers on the left of the "divides" bar. The number in the blank must be the smallest collection of factors that can be divided by both prime factorizations.

Algebra example: Find GCF (think smaller) and LCM (think larger) for these expressions:

$$8a^3xz^2 \qquad 12a^2bx^2z^2$$

____ | $8a^3xz^2$ | ____

| $12a^2bx^2z^2$ |

Greatest Common Factor, Lowest Common Multiple

I. Factors for 150: 1×150, 2×75, 3×50, 5×30, 6×25, 10×15 (twelve in all)
for 80: 1×80, 2×40, 4×20, 5×16, 8×10 (ten of them in all)

II. Review of prime factorization: $150 = 15 \bullet 10 = 2 \bullet 3 \bullet 5^2$
$80 = 8 \bullet 10 = 2^4 \bullet 5$

Prime factorization of $150 = 2 \bullet 3 \bullet 5^2$ can be used to count the 12 factors. $150 = 2^1 \bullet 3^1 \bullet 5^2$ Now add one to *each exponent* of prime factorization, and multiply the results: $(1+1)(1+1)(2+1) = 2 \bullet 2 \bullet 3 = 12$.

III. Choosing between four ways of finding GCF and LCM is dependent on number size, ease of factoring, and how many numbers are given.

A. From lists. List all factors of the two (or more) numbers. Choose the largest one in common. This is the GCF.

For the LCM, list multiples of the given numbers. Choose the smallest number appearing in every list. This is the LCM.

B. Prime factorization with "divides" bar (6|18) per Dr. Dawn Slavens. The number on the left of the "divides" bar is usually smaller than the number on the right.

To find GCF, a *smaller* number, write given numbers on the right of "divides" bars. The number in the blank must be the biggest collection of factors that is contained in (will divide into) both prime factorizations.	____ | $2 \bullet 3 \bullet 5^2$ | ____ ____ | $2^4 \bullet 5$ |	To find LCM, a *larger* number, write given numbers on the left of "divides" bar. The number in the blank must be the smallest collection of factors that contain (can be divided by) both prime factorizations.

C. The Mik method, named for the classroom teacher who mentioned it in a graduate-school class.

) 1 5 0   8 0

Related theorem:

For 150 and 80, $GCF = \dfrac{150 \times 80}{LCM}$

For 91 and 221, $GCF = \dfrac{91 \times 221}{LCM}$

D. Euclidean algorithm and related theorem. Use this when the given numbers are large or otherwise difficult to factor.

$91\overline{)221}$ = 2, 182, remainder 39 → $39\overline{)91}$ = 2, 78, remainder 13 → $13\overline{)39}$ = 3, 39, remainder 0 yes!

$\Rightarrow$ 13 *is GCF*

Parabola Extravaganza

...connecting graph, chart values, factoring, quadratic formula, more...

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1. For $y = f(x) = 3(x+2)^2 - 3$ (A) Vertex coordinates are ____________

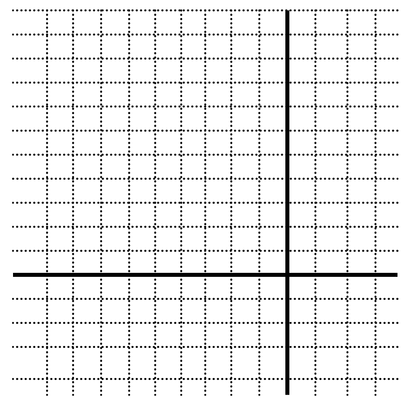
(B) Write vertex coordinates in the MIDDLE of the chart.

(C) Write three consecutive integer values of ***x*** smaller and larger than that of the ***x*** value that you wrote in the chart for the vertex.

(D) Find ***y*** values.

x	***y***

(E) Graph:



(F) Draw and label the axis of symmetry.

(G) Write the equation of the axis of symmetry. ________________

(H) Label the vertex <u>on the graph</u> and <u>in the chart</u>.

(I) FOIL to write $y = f(x) = 3(x+2)^2 - 3$ in $y = f(x) = ax^2 + bx + c$ form.

(J) Factor the results of (I).

(K) Set y = 0 and solve.

2. State the quadratic formula, beginning with "Given..."

$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

3. Complete for $y = f(x)$ $= 3x^2 + 12x + 9$	y-intercept	x-intercepts	axis of symmetry	discriminant (# zeroes)	axis-to-answer distance	vertex
Identifier:	0,?	?,0 ??,0	$x = -\frac{b}{2a}$	$b^2 - 4ac$	$\frac{\pm\sqrt{b^2-4ac}}{2a}$	
$a =$ $b =$ $c =$						

4. Solve $y = f(x) = 3x^2 + 12x + 9$ using the quadratic formula. (haha: "done" already)

Domain and Domain Issues

The *domain of a function* is the set of all first elements of the ordered pairs that constitute the function. Huh?

- A *relation* is a set of ordered pairs: $T = \ -1,3\ ,\ 1,6\ ,\ 3,9\ ,\ x,y\ ,\ 1,7$
- A *function* is a special kind of relation wherein each x or first element is loyal to exactly one y or second element. What pair would we remove in T above to make it a function? (Call the function T_1.)
- The *domain of a function* is the set of all first elements from each ordered pair that comprise the function. For T_1 above, domain =

  $-1,\ 1,\ 3,\ x$

The domain of a function is usually chosen from the set of real numbers. <u>Unless the function involves one of the three issues that follow, the domain **is** the set of real numbers</u>. This set is often designated by $\mathfrak{R}$.

<u>**The three issues**</u> are:

1. Division by zero is not allowed; denominators must not equal zero.

   Because the denominator of $\frac{7x}{x^2-16}$ includes a variable, the denominator might equal zero.

   We prevent this by writing $x^2 - 16 \neq 0$, so $x^2 \neq 16$, and $x \neq \pm 4$

   So the domain statement is, "x can be any real number except ± 4"

   In less plain English, "x is an element of the set $\mathfrak{R}$ so that $x \neq \pm 4$"

   Also, the domain is the set of all x in $\mathfrak{R}$ so that $x \neq \pm 4$.

   In symbols, the domain = $x \in \mathfrak{R} \mid x \neq \pm 4$

2. Square roots of negative numbers are not allowed. For $\sqrt{n}$, n must be greater than or equal to zero.

   For $\sqrt{1-x}$, $1-x$ must be greater than or equal to zero.

   $1 - x \geq 0$

   $-x \geq -1$. domain = $x \in \mathfrak{R} \mid x \leq 1$

   $x \leq 1$

3. Logarithms only exist for positive numbers. For $\log_b n$, $n > 0$.

   For $\log_b\ 2y+10$, $2y + 10 > 0$, so $y > -5$. Domain: $y \in \mathfrak{R} \mid y > -5$

Again, <u>domain is $\mathfrak{R}$</u> unless you see variables (a) in denominators, (b) inside square roots symbols, or (c) in logarithm arguments. Then <u>DO THE WORK</u>.

Consecutive Integers Reminders

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Consecutive Integers

29	n	-16	x
30	$n+1$	-15	$x+1$
31	$n+2$	-14	$x+2$
32	$n+3$	-13	$x+3$
		-12	$x+4$

Consecutive **<u>Odd</u>** Integers

39	a	-17	w
41	$a+2$	-15	$w+2$
43	$a+4$	-13	$w+4$
45	$a+6$	-11	$w+6$
		-9	$w+8$

Consecutive **<u>Even</u>** Integers

56	k	-22	q
58	$k+2$	-20	$q+2$
60	$k+4$	-18	$q+4$
62	$k+6$	-16	$q+6$
		-14	$q+8$

non-consecutive uneven more−than−oddly strange patternless integers

NEVER NEVER EVER DO THE FOLLOWING NEVER EVER

x .	110	y	−51
x+1	111	y+1	−50
x+3	113	y+3	−48
x+5	115		

Regular Polygons and Tessellations

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Regular polygons are ___________________ and _______________________.

A regular quadrilateral is a ______________ and a regular three-gon is called what?

The sum of the vertex angles of any polygon: triangulation exercise.

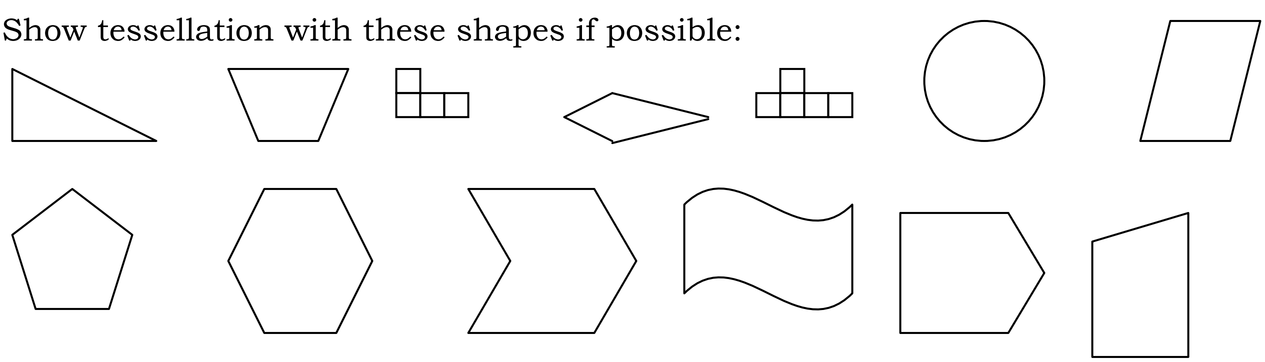
entity	**what is it?**	**sum for any polygon**	**size for regular polygon**
Vertex angle			
Exterior angle			
Central angle			

What is a **polygonal region**?

A **tessellation** is an arrangement of congruent polygons that covers the plane and satisfies two conditions:

(A) (B)

Show tessellation with these shapes if possible:



Which regular polygons will tessellate the plane?

Lateral Surface Area, Total Surface Area, Sphere Area

Prism Volume, Pyramid Volume, Sphere Volume

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(A) Distinguish between *perimeter*, *area*, and *volume*. (B) Review circle attribute schema. (C) Review *polygon*.

A geometric solid can be a polyhedron with sides that are polygons, or a non-polyhedron with at least one side a non-polygon, such as a cylinder.

Polyhedrons are either prisms (lateral sides parallel) or pyramids (all lateral sides meeting at a single point), or pieces thereof. Some pieces (frustums) have had pyramid tops sliced off with the slice parallel to the base. Others are just a mess, like a lump of coal.

Non-polyhedron solids worth studying have curved surfaces.

- The cylinder is the cousin of the prism, because the sides go "straight up" -- which is as hard to define as "between."
- The cone is the cousin of the pyramid, because its lateral side gathers at a point.
- A sphere is the surface of a ball; the sphere is only the shell of a ball; the sphere plus its interior is a ball.

Some non-polyhedron solids are not of interest to mathematicians, unless they study chaos, like an old wad of gum.

In outline form:

A. Polyhedrons: faces are polygons.
   1. Prisms
   2. Pyramids
   (3. Frustums and lumps of coal)

B. Non-polyhedrons
   1. Cylinders
   2. Cones
   3. Spheres
   (4. Old wads of gum)

Lateral Surface Area = area of the sides, not including the bases.
Total Surface Area = area of the sides plus the base or bases.
Sphere Area = area of four circles of the same radius.

Prism Volume = base area × height
Pyramid Volume = one-third base area × height
Sphere Volume = sum of pyramids: one-third $(4\pi r^2 \times r) = \frac{4}{3}\pi r^3$

PRINCIPLES OF NEGATION

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The negation of a statement such as "All birds fly" is roughly formed by saying, "It is false that 'all birds fly.'" These rules also apply:

Rule 1: If a statement is true, its negation must be false.
Rule 2: If a statement is false, its negation must be true.

Below is a table of four statements that share negation relations but in ways that are not discerned easily. Statements in parenthesis are equivalent to those immediately above; "strong" or very clear statements are italicized:

A: All birds fly. (No birds don't fly.)	B: No birds fly (All birds don't fly.)
C: Some birds fly (Not all birds don't fly) (*At least one bird flies*.)	D: Some birds don't fly (Not all birds fly) *(At least one bird doesn't fly)*

The task now is to find the true negation relationships. On the surface, A and B look like the "most opposite", but in mathematics, that's not what is meant by negation. Let's look at statement A carefully and see what we can find to negate it. We'll start by applying rules 1 and 2.

A and B could both be false (on planet Earth, for instance). So to say A and B are negations violates Rule 2 above.

A and C could both be false (if all but the penguins died out, say). So A and C aren't negations of each other, either.

So **A and D <u>are</u> negations of each other**. Someone believing A will always disagree with someone who believes D.

Now look at statement B.
Compared with D, we can see that B and D would both be true in a world of penguins only. So B and D are not negations of each other.
B and C are true negations of each other.

Note patterns of some importance and help:

A and D are negations: A is an "all" statement; D is a "some" statement.
B and C are negations: B is an "all" statement; C is a "some" statement.
A and D are related by "all do" vs. "some don't", and
B and C are related by "none do" vs. "some do."

"All" and "none" are <u>universal quantifiers</u>. "Some", "not all", and "at least one" are <u>existential quantifiers</u>. If a statement involves a universal quantifier, its negation will involve an existential quantifier, and vice versa.

$$a^x = e^{\ln a^x} = e^{x\ln a}$$

Derivatives and Integrals

$\frac{d}{dx}\left[a^u\right] =$	$a^u \ \ln a \ \ u'$		$\frac{d}{dx} \ \log_a u \ =$	$\frac{1}{u\ln a}u'$
⇨ $\frac{d}{dx}\left[e^u\right] =$	$e^u u'$		⇨ $\frac{d}{dx} \ \ln u \ =$	$\frac{1}{u}u'$
⇨ $\frac{d}{dx}\left[a^x\right] =$	$a^x \ln a$	*	⇨ $\frac{d}{dx} \ \log_a x \ =$	$\frac{1}{x\ln a}$
$\frac{d}{dx} \ \tan u \ =$	$\sec^2 u \ \ u'$		$\frac{d}{dx} \ \cot u \ =$	$-\ \csc^2 u \ \ u'$
$\frac{d}{dx} \ \sec u \ =$	$\sec u \tan u \ \ u'$		$\frac{d}{dx} \ \csc u \ =$	$-\ \csc u \cot u \ \ u'$
$\frac{d}{dx} \ \arcsin u \ =$	$\frac{u'}{\sqrt{1-u^2}}$		$\frac{d}{dx} \ \arccos u \ =$	$\frac{-u'}{\sqrt{1-u^2}}$
$\frac{d}{dx} \ \arctan u \ =$	$\frac{u'}{1+u^2}$		$\frac{d}{dx} \ \operatorname{arccot} u \ =$	$\frac{-u'}{1+u^2}$
$\frac{d}{dx} \ \operatorname{arcsec} u \ =$	$\frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$		$\frac{d}{dx} \ \operatorname{arccsc} u \ =$	$\frac{-u'}{\lvert u\rvert\sqrt{u^2-1}}$
Definition of the natural log function			$\ln x =$	$\int_1^x \frac{1}{t}\,dt$
Definition of *e*			$\int_1^e \frac{1}{t}\,dt = 1$	
Log Rule for Integration			$\int \frac{1}{u}\,du =$	$\ln\lvert u\rvert + C$
Log Rule for Integration (2)			$\int \frac{1}{u}u'\,dx =$	$\ln\lvert u\rvert + C$
Derivative of the natural log function			$\frac{d}{dx} \ \ln x \ = \frac{1}{x}$;	$\frac{d}{dx} \ \ln u \ = \frac{1}{u} \bullet u'$
Derivative of an inverse function $g(x) = f^{-1}(x)$			$g'(x) =$	$\frac{1}{f'(g(x))}$

Integration Rules for Exponential Functions

$\int a^u du =$	$a^u \frac{1}{\ln a} + C$	*	$\int e^u du =$	$e^u + C$
$\int e^x dx =$	$e^x + C$		$\int \frac{1}{u} du =$	$\ln\lvert u\rvert + C$
$\int \tan u \ du =$	$-\ln\lvert\cos u\rvert + C$		$\int \cot u \ du =$	$\ln\lvert\sin u\rvert + C$
$\int \sec u \ du =$	$\ln\lvert\sec u + \tan u\rvert + C$		$\int \csc u \ du =$	$-\ln\lvert\csc u + \cot u\rvert + C$
$\int \sec^2 u \ du =$	$\tan u + C$		$\int \csc^2 u \ du =$	$-\cot u + C$
$\int \sec u \tan u \ du =$	$\sec u + C$		$\int \csc u \cot u \ du =$	$-\csc u + C$
$\int \frac{1}{\sqrt{a^2 - u^2}} du =$	$\arcsin \frac{u}{a} + C$		$\int \frac{1}{a^2 + u^2} du =$	$\frac{1}{a} \arctan \frac{u}{a} + C$
$\int \frac{1}{u\sqrt{u^2 - a^2}} du =$	$\frac{1}{a} \operatorname{arcsec} \frac{\lvert u\rvert}{a} + C$	3 8 5	Unusual Examples -- Possible?	

$\int \frac{dx}{x \ln x} =$	$\int \frac{\ln x}{x} dx =$	$\int \ln x \ dx =$

Unusual Examples -- Possible?

$\int \frac{4 \bullet dx}{x^2 + 9} =$	$\int \frac{4x \bullet dx}{x^2 + 9} =$	$\int \frac{4x^2 \bullet dx}{x^2 + 9} =$

Unusual Examples -- Possible?

$\int \frac{dx}{x\sqrt{x^2 - 1}}$	$\int \frac{xdx}{\sqrt{x^2 - 1}}$	$\int \frac{dx}{\sqrt{x^2 - 1}}$

(472)

Notes:
(A) Atop page one is a "ground-of-all-being" statement, based on definition of logarithm and a theorem: $a^x = e^{\ln a^x} = e^{x \ln a}$

(B) The top row of page one has general formulas. The next two lines have particular cases of these. A student *learning* the top row should be able to *develop* the next two lines easily.

PROBABILITY CLASSIFICATIONS

The typical course in probability and statistics devotes a section or chapter each to mutually exclusive events, independent events, and conditional probability. Distinctions between these may be lost. This outline may help.

From the "Queen of Hearts" Formula, P(A or B) = P(A) + P(B) – **P(A and B)**
or $P(A \cup B) = P(A) + P(B) - \mathbf{P(A \cap B)}$,
we can make correct distinctions for classifying probability situations.

Mutually exclusive events cannot happen at the same time. Example: let

A: The event of being in Idaho at noon next Friday.
B: The event of being in Cuba at noon next Friday.

What is the probability of either event coming true?

Effect on main formula: **P(A and B)** = 0 ; $\mathbf{P(A \cap B)} = 0$

So $P(A \cup B) = P(A) + P(B) - 0 = P(A) + P(B)$

Independent events happen at the same time *by chance*. Example: let

A: The event of a rabbit eating 2 gm. of clover.
B: The event of a 2 gm. meteor striking Mars.

What is the probability of either event coming true?

Effect on main formula: **P(A and B)** = P(A)·P(B) ; $\mathbf{P(A \cap B)} = P(A)\cdot P(B)$

$$P(A \cup B) = P(A) + P(B) - P(A)\bullet P(B)$$

Dependent events happen together due to a cause-effect relationship.

"The happier the campus, the more students want to attend."
(How much more? Somebody needs to investigate…)

Example: let A: The event that more students attend your University.
B: The event that your University is a happy place.

Effect on main formula: **P(A and B)** > 0; **P(A and B)** ≠ P(A)·P(B)

The main formula $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ is still good. The value of $P(A \cap B)$ is obtained by some experience. It is neither zero, like mutually exclusive events, nor is it $P(A)\bullet P(B)$, like independent events.

Conditional probability is the probability that an event will occur given that another related event has already occurred.

Example: Find probability that a chosen poker-deck card is a king K, given that aces and cards numbered two through ten are eliminated.

Most conditional probability questions can be solved by restricting the sample space. This shrinks the denominator.

If needed, here's a formula:

$$P(K \textit{ given } \text{restriction}) = \frac{P(K \,\&\, \textit{restriction})}{P(\textit{restriction})}$$

Properties of Operations and Relations

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	symbol	key	element	binary properties
OPERATIONS	+	add	numbers, functions	<u>associative</u>
	−	subtract		$(a*b)*c = a*(b*c)$
	×	multiply		
	÷	divide		<u>commutative</u>
	∪	union	sets	$a*b = b*a$
	∩	intersection		
	×	Cartesian product		<u>distributive</u>
	− or ~	difference		$a*(b \Diamond c) = a*b \Diamond a*c$
	△	symmetric difference		<u>identity</u>
	∘	composition	functions	a*ident = ident*a = a
	$\sqrt{\ }$	square root*	numbers	<u>inverse</u>
	A' or $\overline{A}$	complement* of A	sets	each a has an a^{-1} so that $a*a^{-1}$ = ident
		*unary -- others are binary.		

Note that only the distributive property above pertains to two operations together. The other four pertain to only one operation at a time.

	symbols	elements
R*E*LATIONS	=, ≠, >, ≥, <, ≤	numbers
	≅, ~	angles, polygons, polyhedra
	⊥, //	lines, rays, segments
	$A \cap B = \varnothing$*, $A \cap B \neq \varnothing$**, $U - A = B$°°, =, ⊆, ⊂, ⊄, $n(A) = n(B)$⁺	sets

properties

<u>reflexive</u> *An element relates (?) to itself.*
5 = 5 (true) 5 > 5 (false)
∠ABD ≅ ∠ABD (true)

<u>symmetric</u> *If a first element relates to a second, then the second relates (?) to the first.*
If ∠A ≅ ∠B, then ∠B ≅ ∠A (true)
If 5 < 7, then 7 < 5 (false)

<u>transitive</u> *If a first element relates to a second and the second relates to a third, then the first relates (?) to the third.*
If A ⊂ B and B ⊂ C, then A ⊂ C (true)
If $\overleftrightarrow{xy} \perp \overleftrightarrow{wz}$ and $\overleftrightarrow{wz} \perp \overleftrightarrow{pq}$, then $\overleftrightarrow{xy} \perp \overleftrightarrow{pq}$ (false)

The (?) notation above indicates a T-F judgment needed for each relation. The "if" premises must be true premises.

If all three properties are true for a relation, then that relation is an ***equivalence relation***.

*Disjoint **Intersecting
°°Complementary ⁺Equivalent