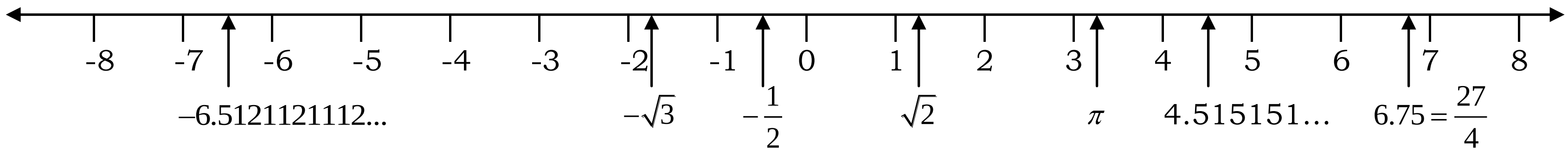
Numbers Outlined

Dr. Stan Hartzler Archer City High School

REAL NUMBERS

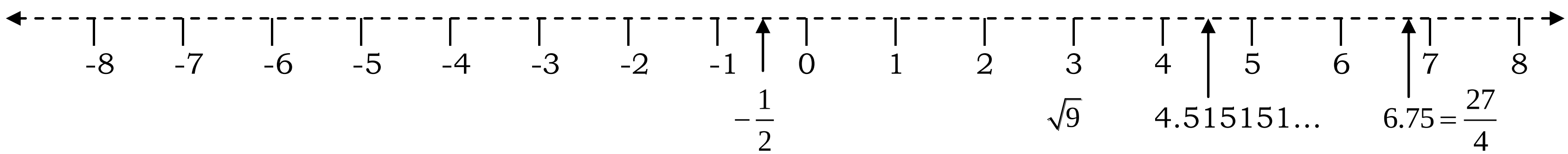
Any without $i = \sqrt{-1}$ as a factor or addend



RATIONAL NUMBERS

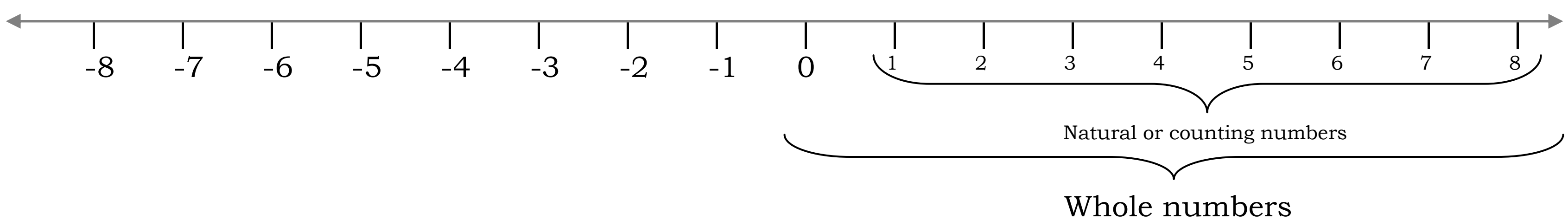
Any that can be written as the ratio of two integers.

[Goodbye to $\sqrt{2}$, π, $-\sqrt{3}$ $-6.5121121112...$]



INTEGERS

(Goodbye to fractions, decimals, percents that can’t be reduced to integers)



Two Takes on Completing the Square

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On the right is some inappropriate review of a process that you learned, and then were told to forget *as* an agency for solving a quadratic equation. The process was mainly used to develop the world's leading method for solving a quadratic equation, namely, the quadratic formula. You were told, less often, that the skills involved would be used elsewhere. On the left is one of those "elsewheres": changing the form of a quadratic *function* as an aid in graphing. This page is an attempt to settle confusion about completing squares, namely,

When do we add to both sides, and when do we add and subtract from *just one side*?

The answer is,

Quadratic function $y = f(x) = ax^2 + bx + c$ in general. $y = f(x) = 2x^2 + 3x + 1$ (specific example) …when re-writing a quadratic function in vertex form. **Add and subtract on the same side** here. $y = f(x) = 2\left(x^2 + \frac{3}{2}x + \left(\frac{1}{2} \bullet \frac{3}{2}\right)^2\right) - 2\left(\frac{1}{2} \bullet \frac{3}{2}\right)^2 + 1$	Quadratic equation: $ax^2 + bx + c = 0$ in general. $2x^2 + 3x + 1 = 0$ (specific example). …when told to solve by completing the square, or… **Add the same thing to both sides** here. … when developing the quadratic formula from the above equation -- you added $-c$ and later $\frac{b^2}{4ac}$ to both sides.

Worth noting:

- Quadratic functions on the left generate many pairs of values x, y and hence the parabola graphs.
- For the specific pairs when $y = 0$, the quadratic function suddenly becomes the quadratic equation on the right. The results are the points where the parabola hits the x axis.
- Thus the equation on the right is a specific subset of what is on the left.
- The graphed quadratic *function* is the entire parabola; the subset of that parabola graph of interest to *equation* solver consists of the points where the parabola hits the x axis.

Greatest Common Factor and Lowest Common Multiple: Making Sense of Prime Factorization

Schema per Dr. Dawn Slavens and MWSU Students

Reminder: Greatest Common Factor is usually **Smaller** than given numbers and Lowest Common Multiple is usually **Larger** than given numbers.

To find GCF and LCM of $\underline{24a^3b^6c}$ and $\underline{36a^4b}$,
rewrite 24 as $2^3 \bullet 3$ and 36 as $2^2 \bullet 3^2$.

To find GCF and LCM of $\underline{2^3 \bullet 3a^3b^6c}$ and $\underline{2^2 \bullet 3^2a^4b}$

GCF smallest collection of each prime factor found in either place

largest collection of each prime factor found in either place LCM

$$\underline{?\qquad\qquad ?}\quad \left| \begin{array}{l} \underline{2^3 \bullet 3 \bullet a^3b^6c} \\ \underline{2^2 \bullet 3^2a^4b} \end{array} \right| \quad \underline{?\qquad\qquad ?}$$

Q: Principles and Formulas for Surface Area and Volume

Area	Linear	***Lateral*** **Surface Area**	***Total*** **Surface Area**	**Volume**
(various formulas)		**Base *Perimeter* × Height** → (rectangle)	Lateral Surface Area + Base Area(s)	prism or cylinder: **Base Area × Height**
A ___	R D P=C __ __ ___	cylinder: can label (rectangle)	cylinder: Dilbert head (rectangle + circles)	pyramid or cone: $\frac{BA \times H}{3}$
		cone: πrL where L = slant height	cone: $\pi rL + \pi r^2$ sphere: $4\pi r^2$	sphere: $\frac{4}{3}\pi r^3$

Slope/Equation/Chart/Graph/Definition Extravaganza

Connecting to… Producing…	Two points as for an *x-y* chart	Equation	Graph of line	Slope = *m*
Starting with… <u>Two points for an *x-y* chart</u>		With the two points in chart, add a third general point (x,y). Write slope two ways, and set those equal. Solve for *y*.	Graph the points and connect with a line.	Chose either first or second *y*; subtract other. Divide result by difference of two *x* values, subtracted in the same order.
Starting with… <u>Equation of line</u>	Make *x-y* chart, select two *x* values, and compute corresponding *y* values.		Generate the two sets of coordinates as described to the left, graph, and connect with a line.	Solve for *y*, writing *x* term and constant term distinctly. Coefficient of *x* is slope *m*.
Starting with… <u>Graph of line</u>	Choose two points on line, and write these in *x-y* chart.	With two points in chart (see cell to left), add general point (x,y). Write slope two ways; set equal. Solve for *y*.		Choose two points on line, and write these in *x-y* chart. Follow instructions in the cell atop this column.
Starting with… <u>Slope = *m** and point P = (x_1,y_1).</u>	Graph point. Move pencil up/down for *y* change, then left/right for *x* change, then mark new point.	Put (x_1,y_1) in chart. Add second general point (x,y). Set $\frac{y-y_1}{x-x_1}$ equal to given *m*. Cross-multiply	Graph point. Move pencil up/down for *y* change, then left/right for *x* change, then mark new point. Connect two points with a line.	

*If *m* is negative, assign negative sign to either numerator or denominator (never both) to start.

A Systematic Look at Conics

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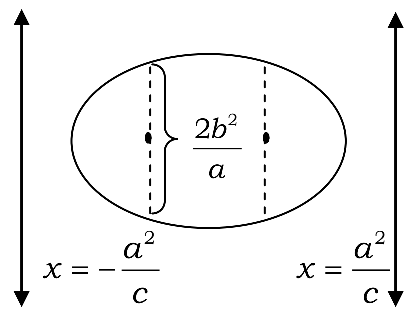
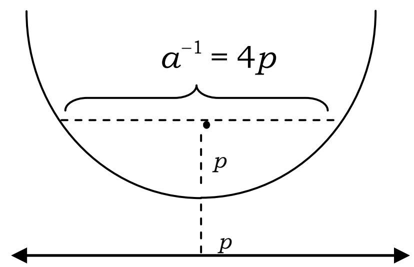
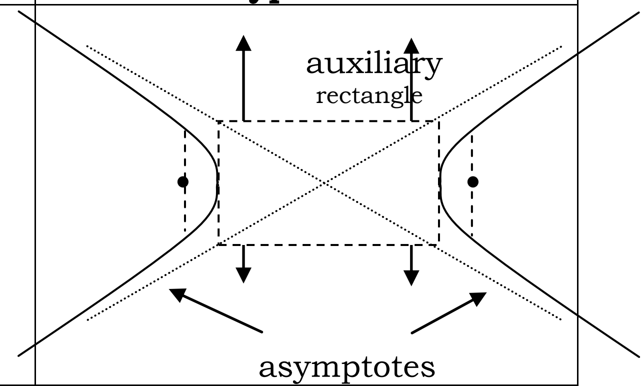
Name / **System**	**Circle**	**Ellipse**	**Parabola**	**Hyperbola**
Eccentricity = e Ratio of focal length to directrix distance	$e =$ 0?	$0 < e < 1$	$e = 1$	$e > 1$
Locus Set of all possible points …	…equidistant from a fixed point (center)	…the sum of whose distances from two fixed points (foci) is constant	…which are equidistant from a point (focus) and a line (directrix)	…the difference of whose distances from two foci is constant
Cone slice	perpendicular to axis of cone	through one nappe, perpendicular to cone axis	parallel to one side of cone	through both nappes of cone
Degenerate	through nappe intersection, producing a point	through nappe intersection, producing a point	coincident with side, producing a line	through nappe intersection, producing two intersecting lines
General Equation	$(x-x_c)^2+(y-y_c)^2=r^2$	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$y=ax^2+bx+c$	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$
degenerate	$r = 0$	$a = b = 0$	$a = 0$	$a = b = 0$

Note that some mathematicians do not agree that a circle is a true conic. Others agree, but dislike the idea that a circle has eccentricity.

Conics Illustrated

a is distance from "center" to farthest point (ellipse) or closest point (hyperbola).
c is distance from center to focus points.

	Ellipse	**Parabola**	**Hyperbola**
illustration			
latus rectum (a segment)	*Latus rectum* is the name of each vertical <u>segment</u> above. The length = $\frac{2b^2}{a}$ of each latus rectum is *focal width*.	*Latus rectum* is the name of the horizontal <u>segment</u> above. The length = a of the latus rectum is the *focal width*.	*Latus rectum* is the name of each vertical <u>segment</u> above. The length = $\frac{2b^2}{a}$ of each latus rectum is the *focal width*.
directrix (a line)	*Directrix* is the name of each <u>line</u> parallel to the lateri recti and outside the ellipse. The parent equations of these are $x=\pm\frac{a^2}{c}$	*Directrix* is the name of the <u>line</u> parallel to the latus rectum the parabola. Its parent equation is $y=p=\frac{1}{4a}$	*Directrix* is the name of each <u>line</u> parallel to the latera recta and outside the hyperbola. The parent equations of these are $x=\pm\frac{a^2}{c}$
vertices	*Vertices* are found at the end points of the major axis.		*Vertices* are where each branch meets the auxiliary rectangle.

All circles and parabolas are similar figures.
All ellipses with the same eccentricity are similar. All hyperbolas with the same eccentricity are similar.

Trigonometric Identities Connections

The Six Basics	Double-Angle	Power-Reducing	Half-angle
$\sin(u+v) = \sin u \cdot \cos v + \sin v \cdot \cos u$ $\sin(u-v) = \sin u \cdot \cos v - \sin v \cdot \cos u$ $\cos(u+v) = \cos u \cdot \cos v - \sin u \cdot \sin v$ $\cos(u-v) = \cos u \cdot \cos v + \sin u \cdot \sin v$ **The tanθ formulas below come from sinθ/cosθ used on the preceding formulas.** $\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$ $\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$	**Use sum formulas; let $u = v$** $\sin 2u = 2\sin u \cos u$ $\cos 2u = \cos^2 u - \sin^2 u$ $= 2\cos^2 u - 1$* $= 1 - 2\sin^2 u$ * $\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$ ***These come from substitution with $\sin^2\theta + \cos^2\theta = 1$**	**From the * formulas in the previous column:** $\sin^2 u = \frac{1-\cos 2u}{2}$ $\cos^2 u = \frac{1+\cos 2u}{2}$ $\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$ * *** From where?**	**Replace u with $\frac{u}{2}$ in previous column, and do square root.** $\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$ $\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$ $\tan\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{1+\cos u}}$

Product-to-Sum	Sum-to-Product	
Multiply third formula above by −1; add result to fourth formula and solve for sin$u$ sin$v$. $\sin u \cdot \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$ **Add third and fourth formulas above and solve for** $\cos u \cdot \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$ **Add first and second formulas above and solve for** $\sin u \cdot \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$ **Multiply second formula above by −1; add result to first formula and solve for cos$u$ sin$v$.** $\cos u \cdot \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$	***Preliminary shoveling:*** **Let $x = u + v$ and $y = u - v$.** Add those two equations: $x + y = 2u$ **so** $u = \frac{x+y}{2}$ Subtract those two equations: $x - y = 2v$ **so** $v = \frac{x-y}{2}$	**Now substitute these into the Product-to-Sum formulas and solve for the sum or difference:** $\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$ $\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$ $\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$ $\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$