IV. K. Algebra GCF LCM all four methods

Greatest Common Factor, Lowest Common Multiple

I. Factors for 150: 1×150, 2×75, 3×50, 5×30, 6×25, 10×15 (twelve in all)
for 80: 1×80, 2×40, 4×20, 5×16, 8×10 (ten of them in all)

II. Review of prime factorization: $150 = 15 \bullet 10 = 2\bullet3\bullet5^2$
$80 = 8 \bullet 10 = 2^4 \bullet 5$

Prime factorization of $150 = 2\bullet3\bullet5^2$ can be used to count the 12 factors.
$150 = 2^1\bullet3^1\bullet5^2$ Now <u>add one to *each exponent*</u> of prime factorization, and multiply the results: $(1+1)(1+1)(2+1) = 2\bullet2\bullet3 = 12$.

III. Choosing between four ways of finding GCF and LCM is dependent on number size, ease of factoring, and how many numbers are given.

A. <u>From lists</u>. List all factors of the two (or more) numbers. Choose the largest one in common. This is the GCF.

For the LCM, list multiples of the given numbers. Choose the smallest number appearing in every list. This is the LCM.

B. <u>Prime factorization</u> with "divides" bar ($6 \vert 18$) per Dr. Dawn Slavens. The number on the left of the "divides" bar is usually smaller than the number on the right.

To find GCF, a *smaller* number, write given numbers on the right of "divides" bars. The number in the blank must be the biggest collection of factors that is contained in (will divide into) both prime factorizations.	$\underline{\qquad} \;\Big\vert\; 2\bullet3\bullet5^2$ $\phantom{\underline{\qquad}} \;\Big\vert\; 2^4 \bullet 5$ $\Big\vert\; \underline{\qquad}$	To find LCM, a *larger* number, write given numbers on the left of "divides" bar. The number in the blank must be the smallest collection of factors that contain (can be divided by) both prime factorizations.

C. <u>The Mik method</u>, named for the classroom teacher who mentioned it in a graduate-school class.

$\big)\ \underline{1\ 5\ 0 \qquad 8\ 0}$

D. <u>Euclidean algorithm</u> and related theorem. Use this when the given numbers are large or otherwise difficult to factor.

$$\begin{array}{r} 2 \\ 91\overline{)221} \\ \underline{182} \\ 39 \end{array} \rightarrow \begin{array}{r} 2 \\ 39\overline{)91} \\ \underline{78} \\ 13 \end{array} \rightarrow \begin{array}{r} 3 \\ 13\overline{)39} \\ \underline{39} \\ 0 \end{array}\ \text{yes!}$$

$\Rightarrow$ 13 *is GCF*

Related theorem:

For 150 and 80, $GCF = \dfrac{150\times80}{LCM}$

For 91 and 221, $GCF = \dfrac{91\times221}{LCM}$