

$$a^x = e^{\ln a^x} = e^{x \ln a}$$

Derivatives and Integrals

$\frac{d}{dx}[a^u] =$	$a^u \ln a \ u'$		$\frac{d}{dx} \log_a u =$	$\frac{1}{u \ln a} u'$
$\Rightarrow \frac{d}{dx}[e^u] =$	$e^u u'$		$\Rightarrow \frac{d}{dx} \ln u =$	$\frac{1}{u} u'$
$\Rightarrow \frac{d}{dx}[a^x] =$	$a^x \ln a$	*	$\Rightarrow \frac{d}{dx} \log_a x =$	$\frac{1}{x \ln a}$
$\frac{d}{dx} \tan u =$	$\sec^2 u \ u'$		$\frac{d}{dx} \cot u =$	$-\csc^2 u \ u'$
$\frac{d}{dx} \sec u =$	$\sec u \tan u \ u'$		$\frac{d}{dx} \csc u =$	$-\csc u \cot u \ u'$
$\frac{d}{dx} \arcsin u =$	$\frac{u'}{\sqrt{1-u^2}}$		$\frac{d}{dx} \arccos u =$	$\frac{-u'}{\sqrt{1-u^2}}$
$\frac{d}{dx} \arctan u =$	$\frac{u'}{1+u^2}$		$\frac{d}{dx} \operatorname{arccot} u =$	$\frac{-u'}{1+u^2}$
$\frac{d}{dx} \operatorname{arcsec} u =$	$\frac{u'}{ u \sqrt{u^2-1}}$		$\frac{d}{dx} \operatorname{arccsc} u =$	$\frac{-u'}{ u \sqrt{u^2-1}}$
Definition of the natural log function			$\ln x =$	$\int_1^x \frac{1}{t} dt$
Definition of e			$\int_1^e \frac{1}{t} dt = 1$	
Log Rule for Integration			$\int \frac{1}{u} du =$	$\ln u + C$
Log Rule for Integration (2)			$\int \frac{1}{u} u' dx =$	$\ln u + C$
Derivative of the natural log function			$\frac{d}{dx} \ln x = \frac{1}{x};$	$\frac{d}{dx} \ln u = \frac{1}{u} \cdot u'$
Derivative of an inverse function $g \ x = f^{-1}(x)$			$g' \ x =$	$\frac{1}{f' \ g \ x}$

Integration Rules for Exponential Functions								
$\int a^u du =$		$a^u \frac{1}{\ln a} + C$		*	$\int e^u du =$		$e^u + C$	
$\int e^x dx =$		$e^x + C$			$\int \frac{1}{u} du =$		$\ln u + C$	
$\int \tan u \ du =$		$-\ln \cos u + C$			$\int \cot u \ du =$		$\ln \sin u + C$	
$\int \sec u \ du =$		$\ln \sec u + \tan u + C$			$\int \csc u \ du =$		$-\ln \csc u + \cot u + C$	
$\int \sec^2 u \ du =$		$\tan u + C$			$\int \csc^2 u \ du =$		$-\cot u + C$	
$\int \sec u \tan u \ du =$		$\sec u + C$			$\int \csc u \cot u \ du =$		$-\csc u + C$	
$\int \frac{1}{\sqrt{a^2 - u^2}} \ du =$		$\arcsin \frac{u}{a} + C$			$\int \frac{1}{a^2 + u^2} \ du =$		$\frac{1}{a} \arctan \frac{u}{a} + C$	
$\int \frac{1}{u\sqrt{u^2 - a^2}} \ du =$		$\frac{1}{a} \arccos \frac{ u }{a} + C$			Unusual Examples -- Possible?			
				3	$\int \frac{dx}{x \ln x} =$	$\int \frac{\ln x}{x} dx =$	$\int \ln x \ dx =$	
				8 5				
Unusual Examples -- Possible?					Unusual Examples -- Possible?			
$\int \frac{4 \bullet dx}{x^2 + 9} =$	$\int \frac{4x \bullet dx}{x^2 + 9} =$	$\int \frac{4x^2 \bullet dx}{x^2 + 9} =$			$\int \frac{dx}{x\sqrt{x^2 - 1}}$	$\int \frac{xdx}{\sqrt{x^2 - 1}}$	$\int \frac{dx}{\sqrt{x^2 - 1}}$	

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Notes:

(A) Atop page one is a “ground-of-all-being” statement, based on

definition of logarithm and a theorem: $a^x = e^{\ln a^x} = e^{x \ln a}$ (B) The top row of page one has general formulas. The next two lines have particular cases of these. A student *learning* the top row should be able to *develop* the next two lines easily.