Fractal Forms From Platonic Solids

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Platonic solids have faces that are regular polygons (all sides and angles within are congruent), and all faces are congruent to all other faces. There are only five such Platonic solids, named for Plato, and these are pictured in the chart.

A "fractaled" tetrahedron may be approximately built in the following way. Drinking straws are used below, but paper folded to make the appropriate-sized solids may be used instead.

(1) The frame for a tetrahedron is made of six drinking straws sewn together at the ends. This is the first generation.

(2) Four more smaller tetrahedra are made in the same way, each edge being half a straw length. These second-stage tetrahedra are sewn to the faces of the original, the vertices of the smaller attached to the midpoints of the edges of the larger. A solid with eight vertices and 24 triangular surfaces emerges.

(3) Twenty-four more even-smaller tetrahedra are now made, each with each edge one-fourth of the original straw length, and the vertices of these are attached to the midpoints of the 24 triangular faces. The shape of a cube becomes apparent.

Theoretically, with this process continued indefinitely, the faces of a cube would become more and more complete. This experience gives the student in elementary grades a glimpse of such notions as *infinity* and *limit*.

This project, and a look at the table, suggests a portfolio-type project for elementary or secondary students. Simply making the table is one project. Having completed the teacher-suggested fractaled-tetrahedron project, the student may become curious as to what a fractaled cube looks like. Successive sizes of water-bombs, each smaller size made of paper that has half the edge of the next-larger size, will do nicely. Wooden cubes from a craft store will glue together well for the same effects.

The remaining three platonic solids also fractal into other Platonic solids, but students should not be told even indirectly that anyone knows this to be true. Paper figures glued together are suggested here. One day, some manufacturer of dice in shapes of Platonic solids may provide mathematics teachers' catalogs with raw material here. Half-sizes will always work, but, again, students should be kept in the dark about that detail also. Other ratios may produce pleasing results other than true Platonic solids.

This fractal relationship between Platonic solids is one of several such relationships. Martin Gardner (1961) has written about the "dual" relationship between the five, as well as how "...they continue to play a colorful role in recreational mathematics." Gardner provides his readers with great possibilities, of course.

One other relationship was accidentally discovered in the process of exploring the fractaling of the solids in the writer's family. The straw tetrahedron was of an ideal size to cover the fractaled cube, with each new vertex concurrent with the midpoint of an edge of the tetrahedron. Another look at the table suggests why, and opens the imagination to other discoveries.

In some programs, practical application must accompany the write-up of portfolio projects, and while students must initiate such ideas, prompts are allowed if mentioned in another context. For projects involving fractaled Platonic solids, a teacher may suggest in another time and context that a class consider drawing a space station or create a design for a chandelier.

True practical applications for fractals exist, of course, including modeling growth of crystals, trees, coral reefs, and other phenomena once thought to be entirely chaotic. These applications may be outside the understanding or interest of students at this grade level, and may be hard to justify in light of the particular kind of fractal process described in this article.

REFERENCES

Gardner, M. The second *Scientific American* book of mathematical puzzles and diversions. New York: Simon and Schuster, 1961.