NON-GEOMETRIC CLASSIFICATION/RELATIONSHIP

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ENTITIES	CLASSIFICATION	RELATIONSHIP
Numbers	natural (counting) whole odd/even prime/composite/neither integer rational algebraic irrational transcendental real imaginary	=, >,< relatively prime multiple of factor of; "divides" inverse opposite reciprocal power of, root of identity
Statements	true/false simple compound conjunction disjunction conditional bi-conditional tautology implication	converse inverse contrapositive negation equivalent
Sets	universal finite/infinite empty/non-empty	disjoint intersecting complementary equal subset equivalent
Subsets	proper/improper	
Operations	unary/binary associative commutative	inverse distributive (over)

A Developmental Beginning and Connection for Proportion, Similarity, Slope, and Tangent (Trig)

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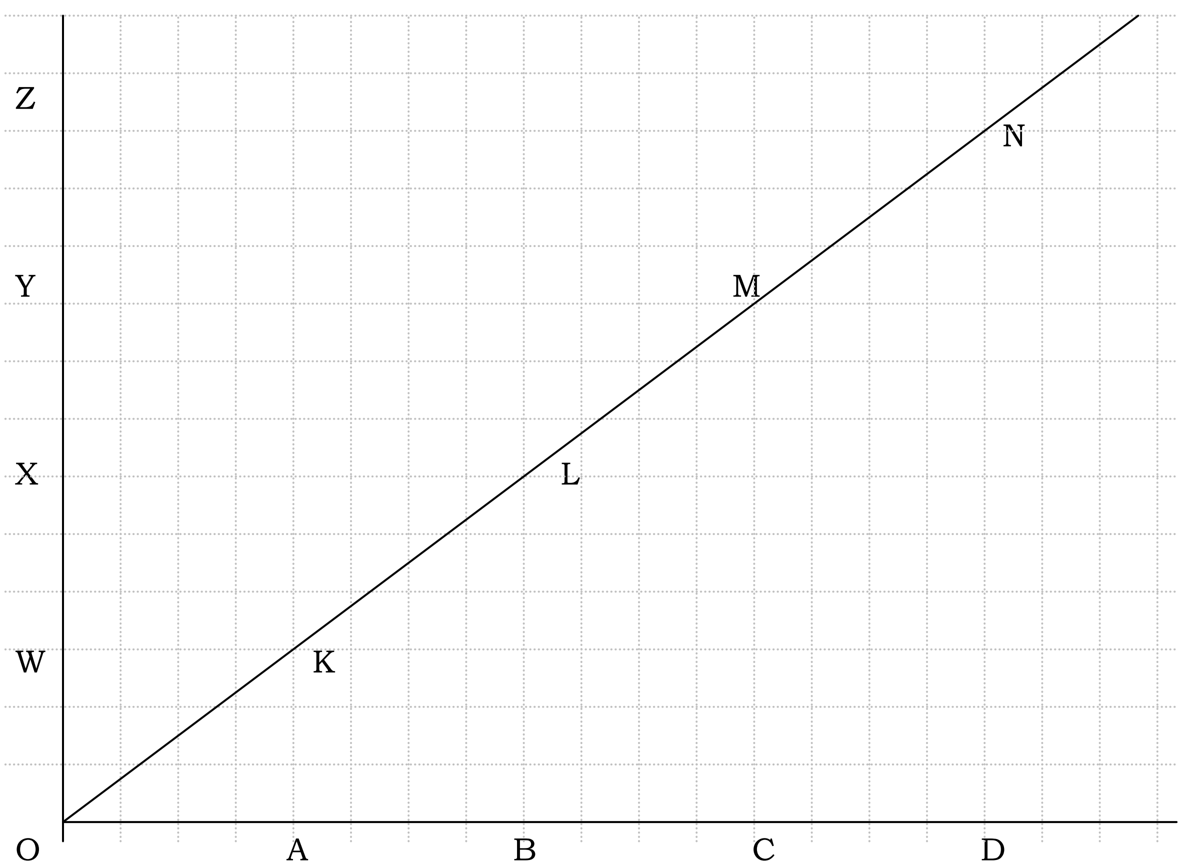
Directions to students:

Find the point named in the chart on the slanted line. Count the number of segments down to the base line. Write that number in the top of the fraction.

Next, go back to the same point on the slanted line. Now count the number of segments needed to go to the side line. Write that number in the bottom of the fraction.

Point name →	Point **K**	Point **L**	Point **M**	Point **N**
Distance down from… / Distance across from… =	______ =	______ =	______ =	______ =
		groups of 2	groups of 3	groups of 4
		______ =	______ =	______ =



Note that

- The equal signs in the chart establish **proportions**
- The arrows reinforce or establish **equivalent fractions**
- The triangles are **similar**
- The $\frac{\text{Distance down from…}}{\text{Distance across from…}}$ fraction is $\frac{\text{change in } y = \Delta y}{\text{change in } x = \Delta x}$ = **slope**
- $\frac{\Delta y}{\Delta x}$ for an angle with vertex at the origin is **tangent** in trigonometry, a cousin of sine and cosine.

The Concept of Denominator/Denomination

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Each of the following is a denominator or denomination, telling the size of a ***uniform quantity*** that is being counted.

- a five-dollar bill
- the 3 in two-thirds
- conventions of place value (for example, the value of the 6 in 365)
- ounces

The purpose of this short paper is to describe connections between these ideas, and to strengthen understanding of each idea.

Cashing a check for $50 at a bank may produce the following question from the teller: "Will two twenties and a ten be all right, or do you need smaller denominations?" Here, denomination refers to the ***quantity*** (size) of money in a given piece of currency or bill.

The denominator 3 in two-thirds tells the ***quantity*** (size) of the fractional part that there are two of. In this sense, the denominator 3 tells amount or size in the same way that denomination of money does.

In the number 9743, the denomination or place value of the 7 is hundreds, by virtue of established ideas of place value. Place value communicates ***quantity*** or size like denomination symbols on currency or denominators of fractions. The same applies to both whole number and decimal place values. The denominations are there but invisible.

The names of units written after numbers establish denominations or ***quantities*** in the same way. Combinations of the above are possible. The denomination of the 7 in “9743 ounces” is *100 ounces*.

Mixed forms of fractions and decimals clarify place value further, and provide an instructive example of the occasional complexity of the concept of denomination. The place value of the 3 in $.005\frac{3}{8}$ is *1/8 of one-thousandth*, or 1/8000. The place value of the 5 is the agreed-on, invisible 1/1000, but the 3/8 is not a place-holder. A fraction carries its own place-value, or denominator.

Like an adjective, a fraction modifies the decimal place immediately to the fraction's left. T he mixed form given above is read, "Five and three-eighths thousandths." (Mixed forms are rarely used but may appear when a student attempts to change a common fraction to a decimal fraction. Vocational classes use these also.) Showing that $66\frac{2}{3}\% = \frac{2}{3}$ will also involve a mixed form.

The denomination of the 3 in $.005\frac{3}{8}$ oz. is *1/8000 oz*.

The main purpose of the above discussion was to exemplify a connection between ideas in mathematics. Two other ideas may be evident.

(A) The ideas of place value, denominator, and denomination are complex; they are studied in graduate schools in mathematics.

(B) There is a great deal of mathematics in arithmetic.

Work with **denominate numbers** helps clarify the notions of denomination. Suppose we are to add the following:

2 ft 8 in + 1 yd 1 ft 9 in

The simplest answer will require carrying a 12-inch foot to the feet column. Re-writing:

		2 ft	8 in
+	1 yd	1 ft	9 in

The similarity between this and carrying tens will help a student with the concept of carrying as well as the concept of denomination. The same applies to subtraction requiring donating {"borrowing") a foot and converting it to 12 inches.

Fractal Forms From Platonic Solids

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Platonic solids have faces that are regular polygons (all sides and angles within are congruent), and all faces are congruent to all other faces. There are only five such Platonic solids, named for Plato, and these are pictured in the chart.

A "fractaled" tetrahedron may be approximately built in the following way. Drinking straws are used below, but paper folded to make the appropriate-sized solids may be used instead.

(1) The frame for a tetrahedron is made of six drinking straws sewn together at the ends. This is the first generation.

(2) Four more smaller tetrahedra are made in the same way, each edge being half a straw length. These second-stage tetrahedra are sewn to the faces of the original, the vertices of the smaller attached to the midpoints of the edges of the larger. A solid with eight vertices and 24 triangular surfaces emerges.

(3) Twenty-four more even-smaller tetrahedra are now made, each with each edge one-fourth of the original straw length, and the vertices of these are attached to the midpoints of the 24 triangular faces. The shape of a cube becomes apparent.

Theoretically, with this process continued indefinitely, the faces of a cube would become more and more complete. This experience gives the student in elementary grades a glimpse of such notions as *infinity* and *limit*.

This project, and a look at the table, suggests a portfolio-type project for elementary or secondary students. Simply making the table is one project. Having completed the teacher-suggested fractaled-tetrahedron project, the student may become curious as to what a fractaled cube looks like. Successive sizes of water-bombs, each smaller size made of paper that has half the edge of the next-larger size, will do nicely. Wooden cubes from a craft store will glue together well for the same effects.

The remaining three platonic solids also fractal into other Platonic solids, but students should not be told even indirectly that anyone knows this to be true. Paper figures glued together are suggested here. One day, some manufacturer of dice in shapes of Platonic solids may provide mathematics teachers' catalogs with raw material here. Half-sizes will always work, but, again, students should be kept in the dark about that detail also. Other ratios may produce pleasing results other than true Platonic solids.

This fractal relationship between Platonic solids is one of several such relationships. Martin Gardner (1961) has written about the "dual" relationship between the five, as well as how "...they continue to play a colorful role in recreational mathematics." Gardner provides his readers with great possibilities, of course.

One other relationship was accidentally discovered in the process of exploring the fractaling of the solids in the writer's family. The straw tetrahedron was of an ideal size to cover the fractaled cube, with each new vertex concurrent with the midpoint of an edge of the tetrahedron. Another look at the table suggests why, and opens the imagination to other discoveries.

In some programs, practical application must accompany the write-up of portfolio projects, and while students must initiate such ideas, prompts are allowed if mentioned in another context. For projects involving fractaled Platonic solids, a teacher may suggest in another time and context that a class consider drawing a space station or create a design for a chandelier.

True practical applications for fractals exist, of course, including modeling growth of crystals, trees, coral reefs, and other phenomena once thought to be entirely chaotic. These applications may be outside the understanding or interest of students at this grade level, and may be hard to justify in light of the particular kind of fractal process described in this article.

REFERENCES

Gardner, M. The second *Scientific American* book of mathematical puzzles and diversions. New York: Simon and Schuster, 1961.

Reasoning: Induction & Deduction

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Induction is the natural thought process by which the mind considers several examples which differ somewhat but have something in common as well, from which the mind draws (induces) a general conclusion or rule. "Start with examples; end with rule." "Specific examples to general conclusion."

Deduction is the natural thought process by which the mind considers an existing rule and one specific example for which the rule applies, and then makes the application. "Start with a rule and one relevant example; apply the rule to the example." "General conclusion to specific example."

Try these.

I. The first time he asks her out, she has to study. The next time, she's tired. The third time, her cousin is coming to town. The next time, she has to wash her hair. The fifth time, she has to finish a paper. (A) What conclusion might he draw? (B) Is this induction or deduction?

A. ____

B. ____________________

II. It is a fact that some species of fish can't pump water over their gills, and must swim constantly to stay alive. A living member of one of those species is filmed in its natural state. (A) What conclusion might we draw? (B) Is this induction or deduction?

A. ____

B. ____________________

III. Conjella identifies several birds flying around Texoma: robin, mockingbird, grackle, scissortail, sparrow, and sparrow hawk. Conjella concludes that all birds fly. Such a conclusion is a *conjecture*, and people make them regularly. Children are especially energetic about making conjectures, as this is how they learn about much of the world. They learn to express these conjectures and test them with adults and older children. "Momma, do all birds fly?" How should a teacher or parent respond to such a conjecture?

IV. In the case of Conjella's conjecture, a *counterexample* is useful. What are some counterexamples to her conjecture?

V. Having inspected squares, rectangles, rhombuses, trapezoids, and parallelograms, Overton concludes that all quadrilaterals have at least two parallel sides. The cognitive psychologists call this over-generalizing. The 1960 college English professor called this hasty generalization. Why? What kind of logic is being attempted?

VI. What kind of logic is "This is the year that the Cubbies go all the way." ?