The Connecting and Distinguishing Concept of Order

Dr. Stan Hartzler Archer City High School

Order matters with respect to counting numbers, integers, place value, names of commas, common fractions, and so on. The following topics make use of the concept of order in other, varying ways:

1. Permutation (vs. combination)

2. $(3,5) \neq (5,3)$ but $\{3,5\} = \{5,3\}$

3. Conditional statement (order of *p* and *q* important); (compared to conjunction, disjunction, biconditional, where order of *p* and *q* is unimportant)

4. Commutative property: order important for subtract and divide, but not for add, multiply, union, intersection.

5. IX and XI -- order of symbols.

6. Find number between .03 and .04

7. Rank-order data to find median and mode

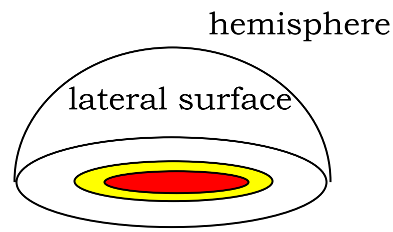
8. Order of multiplication of matrices important (not commutative) (as opposed to number multiplication, which is commutative)

Surface Area Progression

Dr. Stan Hartzler Archer City High School

"Can" refers to a cylinder that contains a sphere of radius r .
The can's height is $2r$ and the base radius is r .

"Cone" is a cone with base radius r and height $2r$.

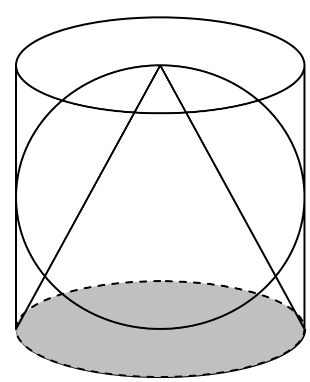


circle	πr^2
hemisphere lateral surface only	$2\pi r^2$
hemisphere total surface area	$3\pi r^2$
sphere	$4\pi r^2$
lateral surface area of can	$4\pi r^2$
lateral surface area of can + bottom	$5\pi r^2$
total surface area of can	$6\pi r^2$
total surface area of cone	$\pi r^2(1+\sqrt{5})$

Volume Progression

Same terminology as above.

cone volume	$\frac{2}{3}\pi r^3$
hemisphere volume	$\frac{2}{3}\pi r^3$
can minus sphere volume	$\frac{2}{3}\pi r^3$
sphere volume	$\frac{4}{3}\pi r^3$
volume of two-tennis-ball can minus ball volumes	$\frac{4}{3}\pi r^3$
can minus cone volume	$\frac{4}{3}\pi r^3$
can volume	$\frac{6}{3}\pi r^3 = 2\pi r^3$
volume of three-tennis-ball can minus ball volumes	$\frac{6}{3}\pi r^3 = 2\pi r^3$



Types of Variation

Dr. Stan Hartzler Archer City High School

Ratio, proportion, and variation should begin in early grades with ratio. Proportion follows naturally with equivalent fractions.

Proportion becomes an important problem-solving tool when students become comfortable with variables and solving simple equations. Applications include similarity in geometry and an amazing array of practical situations in science and engineering -- hence, this topic is included with Rainbows and Starscapes.

Variation includes these:

* *Direct variation*, where increase in one variable means increase in another. Example: if speed is held constant, an increase in distance means an increase in time. Two ideas vary directly if the ordered division of the variables yields a consistent quotient.

Two forms: if K is a constant, $y = Kx$ or $K = \frac{y}{x}$.

* *Inverse variation*, where increase in one variable means decrease in another. Example: if distance is constant, an increase in speed means a decrease in time. Two ideas vary inversely if the product of these variables yields a consistent product.

Forms: $xy = K$ or $x = K/y$.

* *Joint variation* involves a constant K and three variables. If x varies jointly with y and z, then $x = Kyz$ or $K = \frac{x}{yz}$. Example: K might be rate of interest, x is amount of interest, y is principal, and z is time. Another example is similar to the riddle that begins, "If a hen-and-a-half can lay an egg-and-a-half in a day-and-a-half, how…?" Situations involving products or units of work (eggs), workers (hens), and time (days) show joint-variation constancy this way:

$$K = \frac{jobs}{(\text{workers})(\text{time})}$$

* *Combined variation* involves both direct and inverse variation. If x varies directly as the square of y and inversely as z, then

$$x = Ky^2/z \text{ , or } K = \frac{xz}{y^2} .$$

Of the two forms given above in each case, the form that is solved for K is the most convenient to use when solving problems. The other form is more often given in textbooks.

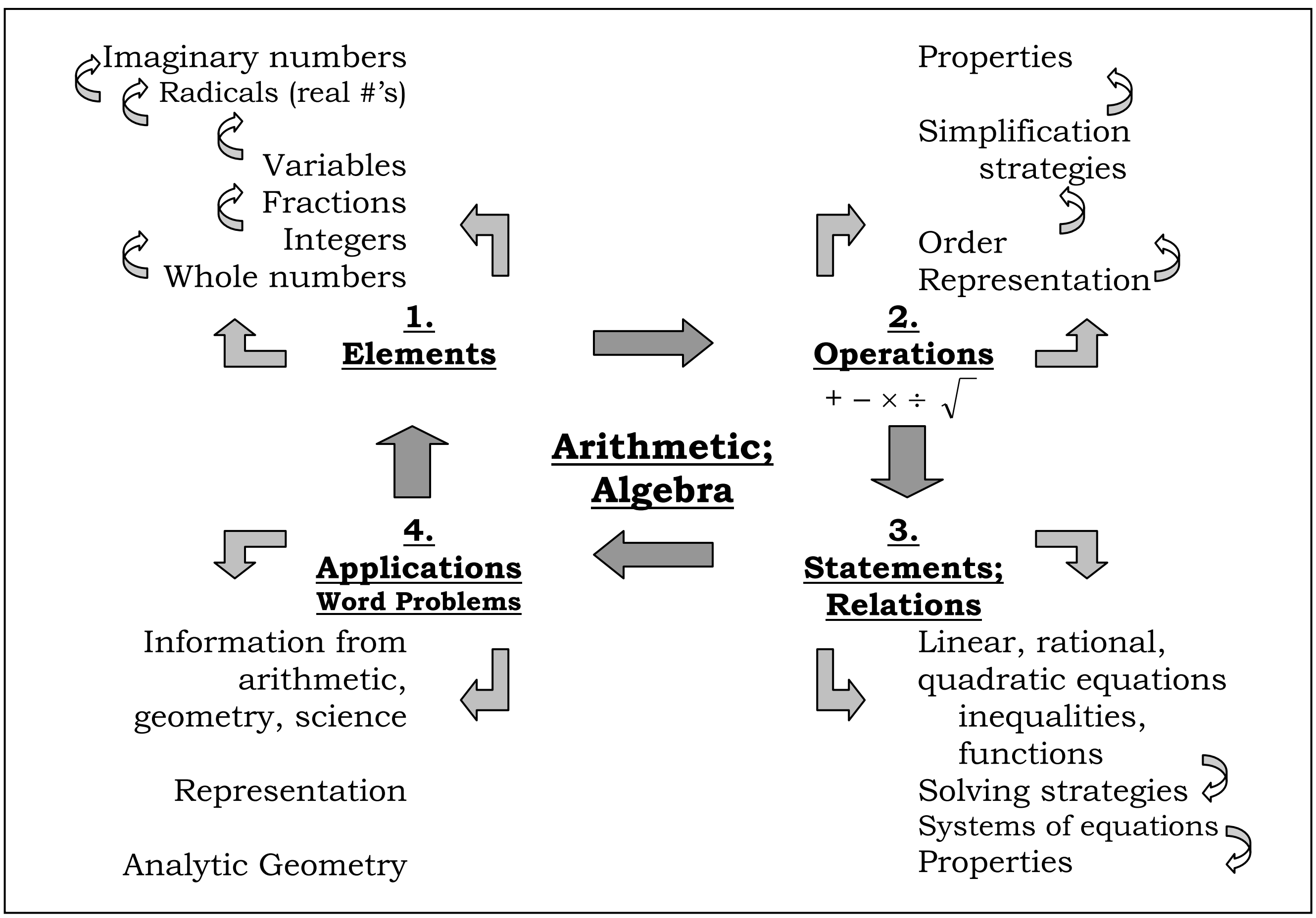
Concept Nodes in Arithmetic and Algebra

Dr. Stan Hartzler   Archer City High School

The concept of *cat* is associated with a written word, a spoken word, images, experiences, cat noises (purr, meow, growl, etc.), kinds of cats, global categories such as animal that include cats, and more. All of these elements contribute to a cat concept node.

Below is an attempt to show why the process of learning mathematics in general, and arithmetic and algebra in particular, is greatly simplified by ongoing review. With ongoing review, connections within and between all the vast nodes are discovered and reestablished daily, whereas drilling daily on but one obscure corner does nothing. The attempt below shows the central node surrounded by four basic arithmetic/algebraic sub-nodes, numbered to direct an endless clockwise-spiraling path. Each sub-node is further subdivided (moving outward) into its own nodes, each incredibly vast. Two examples are attached. The connections between these ideas crisscross endlessly across the page.

Daily review on a widely-varied salad keeps individual ideas activated in memory so that connections and associations can be made. Memory on individual topics is also enhanced, of course.



UPPER-LEVEL DEFINITION CHAIN

Dr. Stan Hartzler   Archer City High School

The writer's experience has included teaching the following in a high school class that included abstract algebra. A better example of a sequence of essential mathematics ideas is hard to imagine.

SET -- A collection. Must be described so that anyone knows exactly what is in it and what isn't (well-defined). *SET* is an uncertain concept because of paradox (Barber of Seville).

CARTESIAN PRODUCT (written A × B, and read "the Cartesian product of set A with set B") -- the set of all possible ordered pairs (x,y) where x is from A and y is from B.

RELATION -- non-empty subset of a Cartesian product. (Any set of ordered pairs).

DOMAIN -- set of all first elements in a relation.

RANGE -- set of all second elements in a relation.

EQUIVALENCE RELATION -- a relation which satisfies reflexive, symmetric, and transitive properties.

FUNCTION -- relation where each domain element has exactly one range element.

BINARY OPERATION -- function whose domain is a Cartesian product and whose range is a set of single elements.

PERMUTATION -- one-to-one function where domain = range.