The Impossible E

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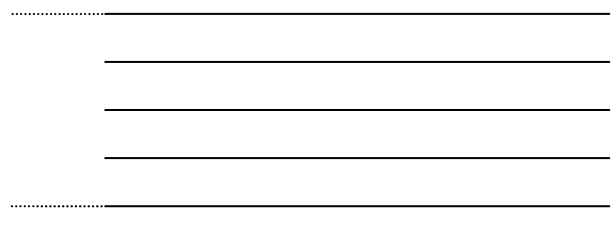
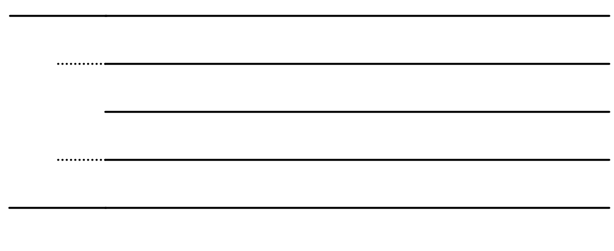
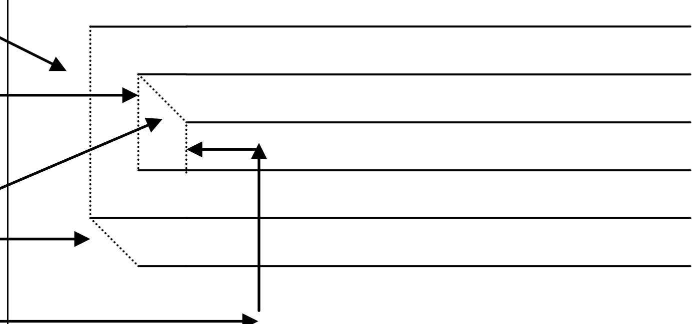
1. Draw six parallel lines, evenly spaced, as shown. The distance between the first and sixth should be less than the length of each.	
2. Estimate the distance between each adjacent pair of lines. Extend the first (topmost) and fifth lines by twice that estimated amount. New steps are dotted.	
3. Again estimate the distance between each adjacent pair of lines. Extend the second, fourth, and sixth lines by that estimated amount.	
4. Connect the left ends of segments 1 and 5. Connect the left ends of segments 2 and 4. With slanted segment, connect ends of 2 and 3, and of 5 and 6. Connect the end of 3 to 4 with segment perpendicular to 4.	
5. Add three ovals to right end as shown. Draw mice or other critters to enhance the alternating representation of space.	

The illusion is apparent if one looks at the left side and sees a visual representation of two "prongs" with a space between them. If the eyes follow the space to the right, that space becomes a solid third prong, between two others. The outer two still look three-dimensional, but for a different reason.

Purposes:
1. Drawing skill
2. Visual estimation
3. Spatial sense
4. Fun
5. Art appreciation (Escher) and problem-solving therein.

Steps for Drawing the Penrose Triangle

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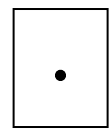
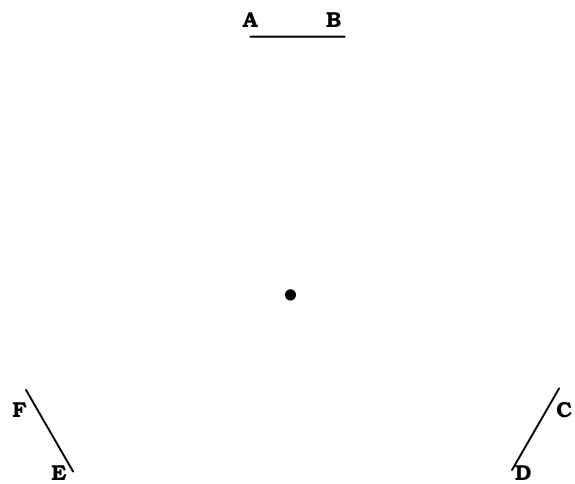
S*patial sense* or *visuospatial sense* is needed for learning mathematics and science. This sense is one of only two of Gardner's "intelligences" that has been confirmed by learning (cognition) scientists. Activities (origami and drawing of ambiguous figures) that help develop spatial sense should be used in classrooms. The purpose of this paper is to provide directions for one such ambiguous drawing.

Visuospatial sense, per Elizabeth Fennema, refers to ability to look at a two-dimensional drawing of a solid object and see it accurately in three dimensions. The mind automatically attempts such sensibility.

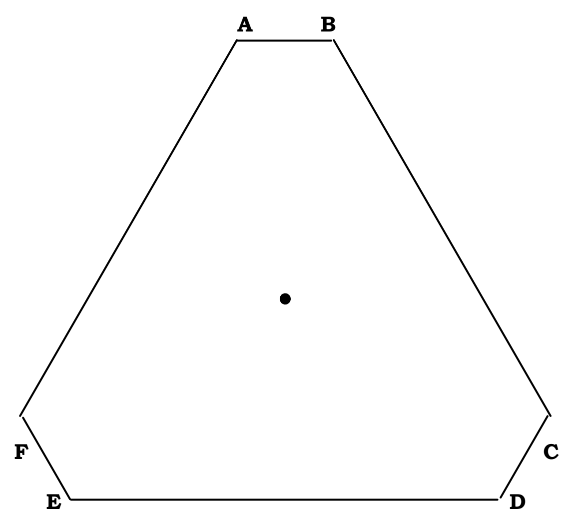
Step 1: Place a dot in the middle of a page of paper, and imagine this to be the center of a clock with hands.



Step 2: Draw $\overline{AB}$, $\overline{CD}$, and $\overline{EF}$ as shown, at positions of 12 o'clock, four o'clock, and eight o'clock, respectively.



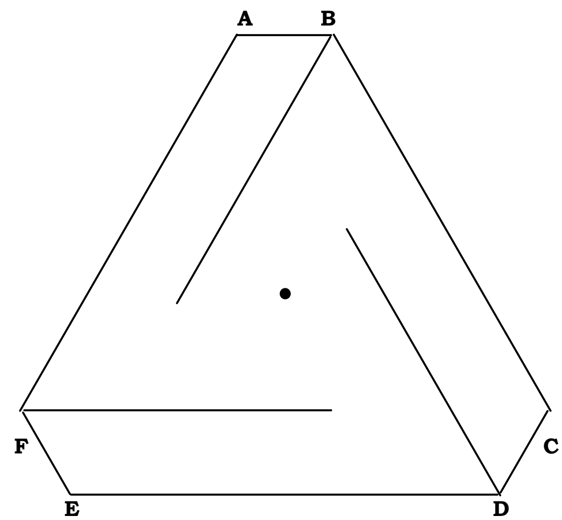
Step 3: Draw $\overline{BC}$, $\overline{DE}$, and $\overline{FA}$ as shown.



Step 4: Place pencil at point F, and begin to draw segment FC. Stop a little past halfway.

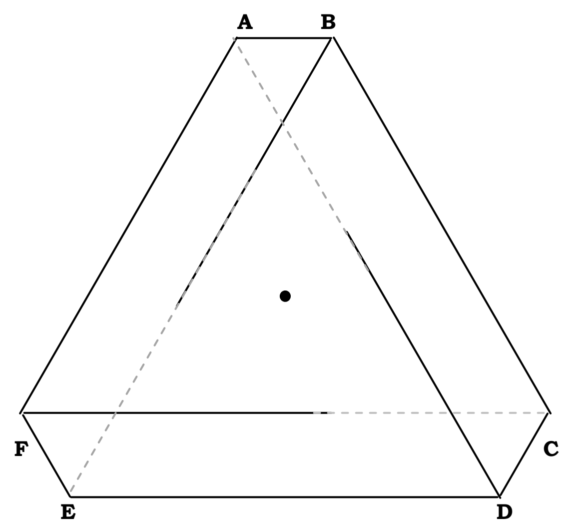
Next, place pencil at point D, and begin to draw segment DA. Stop a little past halfway.

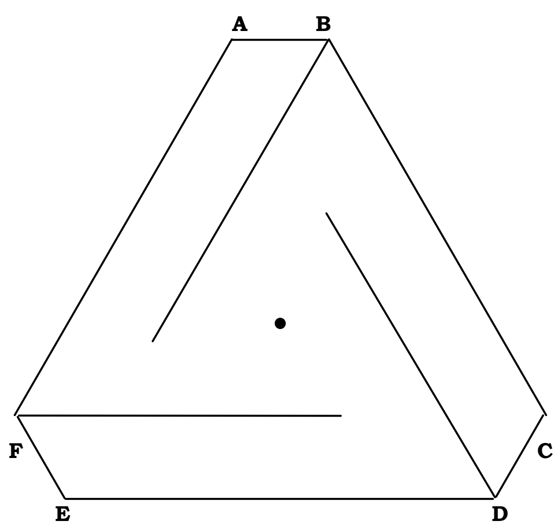
Last, place pencil at point B, and begin to draw segment BE. Stop a little past halfway.



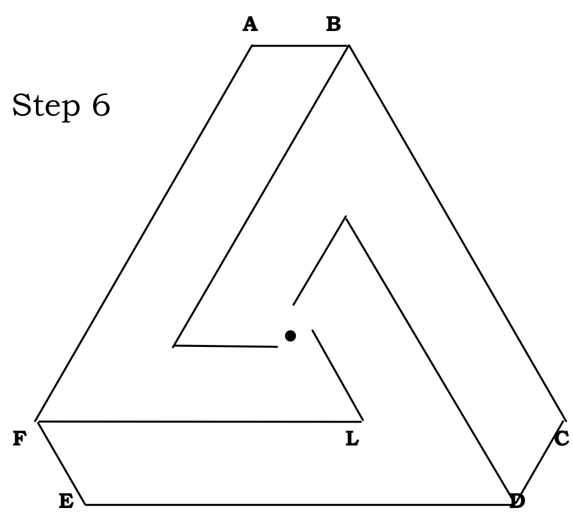
Step 5: Visualize the completion of segment FC. Visualize how much of FC would be between segments CB and DA. That length of FC must be mentally copied on the left of DA. Draw FL so that LC is bisected by DA. Do not draw the dotted line segment LC.

Repeat the same idea for segments DA and BE. Do not draw dotted segments.



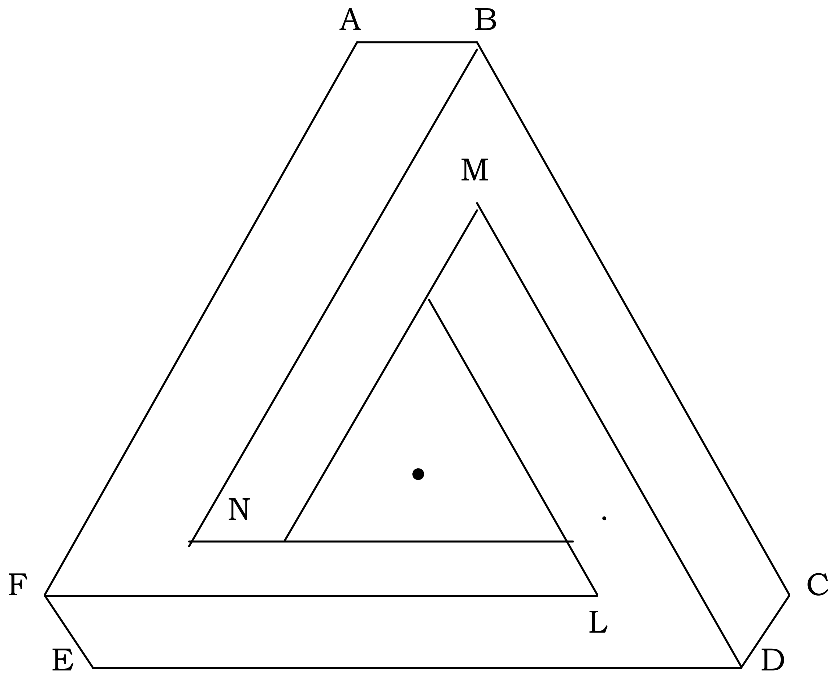


Step 6



Step 6. Place pencil at L and draw a segment parallel to segment CB and partial DA (as much of DA as is drawn) BUT only of a bit past the middle. Repeat for M and N.

Step 7: Continue drawing each of the three new segments until the pencil hits another of the three new segments. The Penrose Triangle is complete.



This triangle suggests a visual paradox. Looking only at the top two "bars" of the triangle, it appears that points A, B, and M (and another hidden point) are included in the intersection of two bars, with *the left-most bar being **in front of** the right-most bar*.

Next, if only the left and bottom bars are examined similarly, it appears that the left bar is behind the bottom bar. By transitivity, this would place *the bottom bar **in front of** the right bar*.

Finally, if only the bottom bar and the right bar are examined, it appears that *the bottom bar is **behind** the right bar*.

Thus, each bar is both in front of *and* behind each of the other two. As eye and mind attempt to make three-dimensional sense of this two-dimensional drawing, frustration will occur without any of this analysis.

Escher artfully involved four such triangles in his graphic, *Waterfall*. Other such impossible figures used by artists include the Impossible E and the Impossible Cage. Such use involves problem-solving…

…as does *Belvedere* by Escher. Steps for drawing the Impossible Cage are on the next page. The main structure in *Belvedere* has supporting columns drawn deceptively, explaining a ladder paradox. On a bench outside the Belvedere, to the viewer's left, a person holds a basic version of the Impossible Cage.

Drawing the Impossible Cage