$$\text{Regular polygon area} = \frac{1}{2}(\text{perimeter} \times \text{apothem})$$

Why? With the center of a regular polygon defined as the point of concurrence of angle bisectors, an *apothem* of a regular polygon is any segment with one end point being the center and the other end point being the midpoint of a side. An apothem is perpendicular to the side intersected, and is an altitude for each isosceles triangle that is formed by connecting the polygon vertices to the center of the regular polygon.

To establish this formula for the area of a regular polygon, students must see that

1) the area of the regular polygon is the sum of the areas of the isosceles triangles, and

2) the *sum of the bases* of these triangles is the *perimeter* of the regular polygon.

At this stage, the **sixth link**, the concept a limit, is introduced. The teacher may show students the following sequence of regular polygons and ask, "As the number of sides gets larger, the regular polygon becomes more and more like what other geometric shape?" Many students can correctly answer that question rather quickly, but (as with all discovery) the teacher must keep everyone silent for at least a few seconds to allow everyone a chance to think.

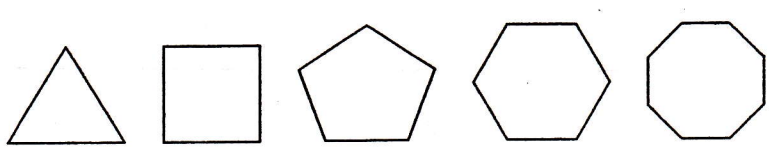

Figure 2

The teacher should also ask, "As the number of sides gets larger, the apothem becomes more like what aspect of that other geometric shape?" The concept of a limit is now used to find the formula for the area of a circle. As the number of sides of a regular polygon increases, the difference between its area and that of a circle containing the regular polygon's vertices decreases. The perimeter of the polygon grows to approximate the circumference of the circle, and the length of the apothem grows to approximate the radius of the circle.

Students thus need to

1) see sectors of a circle as much the same as the isosceles-triangle sections of regular polygons

2) see the area of a circle as the sum of a large number of nearly-triangular sectors of the circle