

3) see the *sum of the bases* of these nearly-triangular-sectors as the circumference or perimeter of the circle

4) establish the altitude of each nearly-triangular-sector as the circle radius.

Making these substitutions into the formula given above for area of a regular polygon gives this exercise in analysis as a **seventh link**:

$$\begin{aligned}\text{Regular polygon area} &= \frac{1}{2} (\text{perimeter})(\text{apothem}) \\ \text{Circle area} &= \frac{1}{2} (\text{circumference})(\text{radius}) \\ &= \frac{1}{2} (\pi \times \text{diameter})(\text{radius}) \\ &= \frac{1}{2} (\pi \times 2r)(r) \\ &= \pi r^2\end{aligned}$$

The analytical experience above requires only confidence with using alphabet letters in formulas, ability to reduce fractions, and reminders that numbers can be multiplied in any order without affecting the result. Nevertheless, this experience, like most of those described in this article, should be briefly repeated or alluded to for several consecutive days to establish student comfort.

Teachers and students should be cautious concerning this type of Extend-to-Limit thinking, as example exist in high school calculus where such approaches fail.

Three Dimensions

Another hands-on activity follows as the **eighth link**, this one from a turn-of-the century geometry textbook. Small groups of no more than three can be used again. Each group needs a solid hemisphere and a length of heavy cord, of perhaps 4mm (5/30s in) thickness, available at hardware or other construction-supply stores.

A variety of hemisphere sizes across the class is desired. The hemisphere can be made from a Styrofoam sphere sold at craft stores.

(An industrial arts professor at Western Kentucky University made some wooden hemispheres from old croquet balls for this writer's methods class, but cautioned that only experts should attempt such wood-working. The risk of serious injury is great when attempting to cut spherical solids with power equipment.)

One end of the cord should be knotted and attached to the top of the hemisphere as illustrated. A deformed paper clip works well with the Styrofoam; an eye-hook eyelet screw is a good choice for wooden hemispheres. The string is wound around the "north-pole" anchor and